Ex:-1 Find the quotient and remainder when $3x^3+8x^2+8x+12$ is divided by $x-4$

Soln

$x-4=0$

$x=4$

4	3	8	8	12
	0	12	80	352
	3	20	88	364

∴ The quotient is $3x^2+20x+88$

Remainder is 364

Ex:-2 Find the quotient and remainder when $2x^6+3x^5-15x^2+2x-4$ is divided by $x+5$

Soln

$x+5=0$

$x=-5$

-5	2	3	0	0	-15	2	-4
	0	-10	35	-175	875	-4300	21490
	2	-7	35	-175	860	-4298	21486

∴ The quotient is $2x^5-7x^4+35x^3-175x^2+860x-4298$

Remainder is 21486.

Ex:-5. S.T the Equation $x^4-3x^3+4x^2-2x+1=0$ can be transformed into a reciprocal Equation by diminishing the roots by unity. Hence Solve the Equation.

Soln

1	1	-3	4	-2	1
	0	1	-2	2	0
1	1	-2	2	0	
	0	1	-1	1	
1	1	-1	1		
	0	1	0		
1	1	0			
	0	1			
	1				

∴ The transformed Equation is

$x^4+x^3+x^2+x+1=0$.

Which is reciprocal Equation

Removal of terms

3. Solve the following equation by removing the 2nd term in each:-

i) $x^4 - 12x^3 + 48x^2 - 72x + 35 = 0$

Soln

Let us assume that by diminishing the roots by h, the second term is removed.

Then the transformed equation becomes

$$(x+h)^4 - 12(x+h)^3 + 48(x+h)^2 - 72(x+h) + 35 = 0$$

$$(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4) - 12(x^3 + 3x^2h + 3xh^2 + h^3) + \cdots = 0$$

$$x^4 + (4h - 12)x^3 + \cdots = 0$$

$$4h - 12 = 0$$

$$4h = 12$$

$$\boxed{h = 3}$$

3	1	-12	48	-72	35
	0	3	-27	63	-27
	1	-9	21	-9	8
	0	3	-18	9	
	1	-6	3	0	
	0	3	-9		
	1	-3	-6		
	0	3			
	1	0			

$$x^4 - 6x^2 + 8 = 0$$

put $x^2 = t$

$$t^2 - 6t + 8 = 0$$

$$(t-4)(t-2) = 0$$

$$t = 4, 2$$

$-6 \mid 8$

$-4-2 \mid -4 \times -2$

$x^2 = 4, \quad x^2 = 2$

$x = \pm 2 \quad x = \pm\sqrt{2}$

∴ The roots of the original equation are

$2+3, \; -2+3, \; \sqrt{2}+3, \; -\sqrt{2}+3$

$5, \; 1, \; \sqrt{2}+3, \; -\sqrt{2}+3$

Descartes' Rule of Signs

An Equation $f(x)=0$ cannot have more positive Then There are changes of sign in $f(x)$.

Descartes' rule of signs for -ve Roots

No Equation can have a greeeber number of negative roots Then There are changes of sign in The terms of the polynomial $f(-x)$.

Ex:-52

S.T The Equation $x^7-3x^4+2x^3-1=0$ has atleast four imaginary roots.

Soln $f(x) = x^7-3x^4+2x^3-1$

$f(-x) = -x^7-3x^4-2x^3-1$

		No. of sign changes
$f(x)$	+ − + −	3
$f(-x)$	− − − −	0

Total No. of Roots = 7

No. of +ve real roots = 3

No. of -ve real roots = 0

No. of Imagenary roots = 4.

2. S.T $x^6 + 3x^2 - 5x + 1 = 0$ has atleast four imaginary roots.

Soln $f(x) = x^6 + 3x^2 - 5x + 1$

$f(-x) = x^6 + 3x^2 + 5x + 1$

					No. of sign changes
$f(x)$	+	+	−	+	2
$f(-x)$	+	+	+	+	0

Total No. of roots = 6

No. of +ve real roots = 2

No. of −ve real roots = 0

No. of Imaginary roots = 6 − 2 = 4

via

3. Find the number of real roots of the Equation $x^3 + 18x - 6 = 0$

Soln $f(x) = x^3 + 18x - 6$

$f(-x) = -x^3 - 18x - 6$

				No. of sign changes
$f(x)$	+	+	−	1
$f(-x)$	−	−	−	0

Total No. of roots = 3

No. of +ve real root = 1

No. of −ve real root = 0

No. of imaginary roots = 3 − 1 = 2.

4. P.T the Equation $x^4 + 3x - 1 = 0$ has two real and two imaginary roots

Soln $f(x) = x^4 + 3x - 1$

$f(-x) = x^4 - 3x - 1$

				No. of sign changes
$f(x)$	+	+	−	1
$f(-x)$	+	−	−	1

Total No. of roots = 4

No. of +ve real roots = 1

No. of −ve real roots = 1

No. of Imaginary roots = 4 − 2

= 2.