

Ex:-1 Find the quotient and remainder when $3x^3 + 8x^2 + 8x + 12$ is divided by $x - 4$

Soln

$$\begin{array}{r|rrrr} 4 & 3 & 8 & 8 & 12 \\ & 0 & 12 & 80 & 352 \\ \hline & 3 & 20 & 88 & 364 \end{array}$$

$$x - 4 = 0$$

$$x = 4$$

$\therefore$ The quotient is $3x^2 + 20x + 88$

Remainder is 364

n.w

Ex:-2 Find the quotient and remainder when $2x^6 + 3x^5 - 15x^2 + 2x - 4$ is divided by $x + 5$

Soln

$$\begin{array}{r|rrrrrrrr} -5 & 2 & 3 & 0 & 0 & -15 & 2 & -4 \\ & 0 & -10 & 35 & -175 & 875 & -4300 & 21490 \\ \hline & 2 & -7 & 35 & -175 & 860 & -4298 & 21486 \end{array}$$

$$x + 5 = 0$$

$$x = -5$$

$\therefore$ The quotient is $2x^5 - 7x^4 + 35x^3 - 175x^2 + 860x - 4298$

Remainder is 21486

Q.1 Q.2 Q.3 Q.4 Q.5 Q.6 Q.7 Q.8 Q.9 Q.10 Q.11 Q.12 Q.13 Q.14 Q.15 Q.16 Q.17 Q.18 Q.19 Q.20 Q.21 Q.22 Q.23 Q.24 Q.25 Q.26 Q.27 Q.28 Q.29 Q.30 Q.31 Q.32 Q.33 Q.34 Q.35 Q.36 Q.37 Q.38 Q.39 Q.40 Q.41 Q.42 Q.43 Q.44 Q.45 Q.46 Q.47 Q.48 Q.49 Q.50 Q.51 Q.52 Q.53 Q.54 Q.55 Q.56 Q.57 Q.58 Q.59 Q.60 Q.61 Q.62 Q.63 Q.64 Q.65 Q.66 Q.67 Q.68 Q.69 Q.70 Q.71 Q.72 Q.73 Q.74 Q.75 Q.76 Q.77 Q.78 Q.79 Q.80 Q.81 Q.82 Q.83 Q.84 Q.85 Q.86 Q.87 Q.88 Q.89 Q.90 Q.91 Q.92 Q.93 Q.94 Q.95 Q.96 Q.97 Q.98 Q.99 Q.100 Q.101 Q.102 Q.103 Q.104 Q.105 Q.106 Q.107 Q.108 Q.109 Q.110 Q.111 Q.112 Q.113 Q.114 Q.115 Q.116 Q.117 Q.118 Q.119 Q.120 Q.121 Q.122 Q.123 Q.124 Q.125 Q.126 Q.127 Q.128 Q.129 Q.130 Q.131 Q.132 Q.133 Q.134 Q.135 Q.136 Q.137 Q.138 Q.139 Q.140 Q.141 Q.142 Q.143 Q.144 Q.145 Q.146 Q.147 Q.148 Q.149 Q.150 Q.151 Q.152 Q.153 Q.154 Q.155 Q.156 Q.157 Q.158 Q.159 Q.160 Q.161 Q.162 Q.163 Q.164 Q.165 Q.166 Q.167 Q.168 Q.169 Q.170 Q.171 Q.172 Q.173 Q.174 Q.175 Q.176 Q.177 Q.178 Q.179 Q.180 Q.181 Q.182 Q.183 Q.184 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Soln

$$\begin{array}{r|rrrrr} 1 & 1 & -3 & 4 & -2 & 1 \\ & 0 & 1 & -2 & 2 & 0 \\ \hline & 1 & -2 & 2 & 0 & 1 \\ & 0 & 1 & -1 & 1 & 0 \\ \hline & 1 & -1 & 1 & 1 & 1 \\ & 0 & 1 & 0 & 0 & 0 \\ \hline & 1 & 0 & 1 & 1 & 1 \\ & 0 & 1 & 0 & 0 & 0 \\ \hline & 1 & 0 & 1 & 1 & 1 \\ & 0 & 1 & 0 & 0 & 0 \\ \hline & 1 & 0 & 1 & 1 & 1 \\ &$$

Removal of terms

3. Solve the following equation by removing the 2nd term in each:-

i) $x^4 - 12x^3 + 48x^2 - 72x + 35 = 0$

Soln

Let us assume that by diminishing the roots by h , the second term is removed.

Then the transformed equation becomes

$$(x+h)^4 - 12(x+h)^3 + 48(x+h)^2 - 72(x+h) + 35 = 0$$

$$(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4) - 12(x^3 + 3x^2h + 3xh^2 + h^3) + \dots = 0$$

$$x^4 + (4h - 12)x^3 + \dots = 0$$

$$4h - 12 = 0$$

$$4h = 12$$

$$\boxed{h = 3}$$

$$\begin{array}{r|rrrrr} 3 & 1 & -12 & 48 & -72 & 35 \\ & 0 & 3 & -27 & 63 & -27 \\ \hline & 1 & -9 & 21 & -9 & 8 \\ & 0 & 3 & -18 & 9 & \\ \hline & 1 & -6 & 3 & 0 & \\ & 0 & 3 & -9 & & 0 \\ \hline & 1 & -3 & -6 & & \\ & 0 & 3 & & & \\ \hline & 1 & 0 & & & \end{array}$$

$$x^4 - 6x^2 + 8 = 0$$

put $x^2 = t$

$$t^2 - 6t + 8 = 0 \quad \begin{array}{r|l} -6 & 8 \\ -4 & -2 \end{array}$$

$$(t-4)(t-2) = 0$$

$$t = 4, 2$$

$$x^2 = 4, \quad x^2 = 2$$

$$x = \pm 2, \quad x = \pm \sqrt{2}$$

$\therefore$ The roots of the original equation are

$$2+3, -2+3, \sqrt{2}+3, -\sqrt{2}+3, 5, 1, \sqrt{2}+3, -\sqrt{2}+3$$

Descartes' Rule of Signs

An equation $f(x)=0$ cannot have more positive roots than there are changes of sign in $f(x)$.

Descartes' rule of signs for -ve roots

No equation can have a greater number of negative roots than there are changes of sign in the terms of the polynomial $f(-x)$.

Ex: -52

S.T. The equation $x^7 - 3x^4 + 2x^3 - 1 = 0$ has atleast four imaginary roots.

Soln

$$f(x) = x^7 - 3x^4 + 2x^3 - 1$$

$$f(-x) = -x^7 - 3x^4 - 2x^3 - 1$$

$$f(x) \quad + \quad - \quad + \quad -$$

$$f(-x) \quad - \quad - \quad - \quad -$$

No. of sign changes
3

0

Total No. of Roots = 7

No. of +ve real roots = 3

No. of -ve real roots = 0

No. of Imaginary roots = 4.

2. S.T $x^6 + 3x^2 - 5x + 1 = 0$ has at least four imaginary roots.

Soln

$$f(x) = x^6 + 3x^2 - 5x + 1$$

$$f(-x) = x^6 + 3x^2 + 5x + 1$$

$$f(x) \quad + \quad + \quad - \quad + \quad \text{No. of sign changes} \quad 2$$

$$f(-x) \quad + \quad + \quad + \quad + \quad 0$$

$$\text{Total No. of roots} = 6$$

$$\text{No. of +ve real roots} = 2$$

$$\text{No. of -ve real roots} = 0$$

$$\text{No. of Imaginary roots} = 6 - 2 = 4$$

wa
3.

Find the number of real roots of the equation $x^3 + 18x - 6 = 0$

Soln

$$f(x) = x^3 + 18x - 6$$

$$f(-x) = -x^3 - 18x - 6$$

$$f(x) \quad + \quad + \quad - \quad \text{No. of sign changes} \quad 1$$

$$f(-x) \quad - \quad - \quad - \quad 0$$

$$\text{Total No. of roots} = 3$$

$$\text{No. of +ve real root} = 1$$

$$\text{No. of -ve real root} = 0$$

$$\text{No. of imaginary roots} = 3 - 1 = 2$$

4. P.T The Equation $x^4 + 3x - 1 = 0$ has two real and two imaginary roots

Soln $f(x) = x^4 + 3x - 1$

$$f(-x) = x^4 - 3x - 1$$

		No. of sign changes
$f(x)$	+ + -	1
$f(-x)$	+ - -	1

$$\text{Total No. of roots} = 4$$

$$\text{No. of +ve real roots} = 1$$

$$\text{No. of -ve real roots} = 1$$

$$\begin{aligned}\text{No. of Imaginary roots} &= 4 - 2 \\ &= 2.\end{aligned}$$