1) S.T $18!+1$ is [illegible]

$437 = 19 \times 23.$

19 & 23 are primes by W.T

$(19-1)!+1$ is divisible by 19.

$18!+1$ is divisible by 19.

$18!+1 \equiv 0 \pmod{19}$

23 is also prime by W.T

$(23-1)!+1$ is divisible by 23.

22 is also prime by W.T.

$22!+1 \equiv 0 \pmod{23}$

$22 \cdot 21 \cdot 20 \cdot 19 \cdot 18!+1 \equiv 0 \pmod{23}$

$(23-1)(23-2)(23-3)(23-4)(23-5)\ 18!+1 \equiv$ (mod

$(-1)(-2)(-3)(-4)\ 18!+1 \equiv 0 \pmod{23}$

$24 \cdot 18!+1 \equiv 0 \pmod{23}$

$(23+1)\ 18!+1 \equiv 0 \pmod{23}$

$23(18!+1)+1(18!+1) \equiv 0 \pmod{23}$

$23(18!+1)$ is divisible by 23.

$18!+1 \equiv 0 \pmod{23}.$

$\therefore 18!+1 \equiv 0$

State & Proof Fermat's

Statement

Let P is prime & a is any number prime to P then $a^{P-1}-1$ is divisible by P.

Proof.

If 'n' is a prime number then ^nC_r divisible by n.

$$(a+1)^P = a^P + PC_1 a^{P-1} + PC_2 a^{P-2} + \cdots + PC_{P-1} a + 1$$

$$(a+1)^P = a^P + PC_1 a^{P-1} + PC_2 a^{P-2} + \cdots\cdots + PC_{P-1} a + 1$$

$$(a+1)^P = a^P + PC_1 a^{P-1} + PC_2 a^{P-2} + \cdots\cdots + PC_{P-1} a + 1$$

$$(a+1)^P = (a^P + 1) + PC_1 a^{P-1} + PC_2 a^{P-2} + \cdots\cdots + PC_{P-1} a$$

$$(\because PC_P = 1)$$

$$(a+1)^P - (a^P+1) = PC_1 a^{P-1} + PC_2 a^{P-2} + \cdots\cdots + PC_{P-1} a$$

PC_1, PC_2, $\cdots$ PC_{P-1} is divisible by P.

$$(a+1)^P - (a^P+1) = \text{a multiple of } P \text{ (M)}$$

$$(a+1)^P - (a^P+1) \equiv 0 \pmod{P}$$

$$\therefore (a+1)^P \equiv (a^P+1) \pmod{P}$$

Since this result is ~~true~~ for all values of a.

2). 3^{100} is divisible by 101.

Sol

$3^6 = 729$

$\equiv 729 \pmod{101}$.

$\equiv 22 \pmod{101}$.

$(3^6)^2 \equiv 484 \pmod{101}$

$3^{12} \equiv 80 \pmod{101} \longrightarrow (1)$.

$(3^{12})^2 \equiv (80)^2 \pmod{101}$

$3^{24} \equiv 6400 \pmod{101}$.

$\equiv 37 \pmod{101}$. $\longrightarrow (2)$.

$(3^{24})^2 \equiv (37)^2 \pmod{101}$.

$\equiv 1369 \mod 101$.

$3^{48} \equiv 56 \mod 101. \longrightarrow (3)$

$(3^{48})^2 \equiv (56)^2 \mod 101$.

$3^{96} \equiv 3136 \mod 101$.

$\equiv 5 \mod 101$.

$3^{96} \cdot 3^4 \equiv 81 \cdot 5 \mod 101$

$\equiv 405 \mod 101$

$3^{100} \equiv 1 \pmod{101}$.

$\therefore$ $n(n+1)(2n+1)$ is divisible by 6.

Note:

i) $n(n+1)(2n+1)$ is the multiple of 6.
This is usually written as $n(n+1)(2n+1) = m(6)$.

ii). The product of r consective integers is divisible by r!.

தொடர்ச்சியான

Show that $n(n^2-1)(29n^2+4) = M(120)$.

Sol

$$n(n^2-1)(29n^2+4) = \cancel{M(120)}$$

$$= n(n^2-1)(29n^2+120-116)$$

$$= n(n^2-1)(29n^2-116+120)$$

$$= n(n^2-1)(29n^2-116) + 120n(n^2-1).$$

$120n(n^2-1). \quad = M(120) \longrightarrow (2).$

consider,

$$n(n^2-1)(29n^2-116).$$

$$29n(n^2-1)(n^2-4).$$

$$29n(n^2-1)(n^2-2^2).$$

$$29n(n+1)(n-1)(n+2)(n-2)$$

$$29(n-2)(n-1)n(n+1)(n+2).$$

Since r is consective integer divisor by r!

$29(n-2)(n-1)n(n+1)(n+2)$ is divisible by 5!

$\left[\frac{1000}{7}\right] = 142$ $\left[\frac{1000}{11}\right] = 90$

$\left[\frac{142}{7}\right] = 20$ $\left[\frac{90}{11}\right] = 8$

$\left[\frac{20}{7}\right] = 2$

The highest power of 7 in 1000!
$142+20+2=164$.

The highest power of 11 in 1000!
$90+8 = 98$.

$\left[\frac{1000}{13}\right] = 76$

$\left[\frac{76}{13}\right] = 5.$

$81.$

The highest power of 13 in 1000! is
$76+5 = 81$.

Show that $n(n+1)(2n+1)$ is divisible by 6

Sol

$n(n+1)(2n+1) = n(n+1)(2n+4-3).$

$= n(n+1)(2n+4) - 3n(n+1).$

$= 2n(n+1)(n+2) - 3n(n+1.$

either n (or) $n+1$ divisible 2

$\therefore 3n(n+1)$ divisible 6.

III'ly

n (or) $n+1$ (or) $n+2$ divisible 3.

$2n(n+1)(n+2)$ is divisible by 6.

The highest power of 3 in 1000!

$333 + 111 + 37 + 12 + 4 + 1 = 498$

Thus 3^{498} is the highest power of 3 dividing 1000!

Find the highest power of 2, 5, 7, 11, 13 contained in 1000!

$\left[\frac{1000}{2}\right] = 500$

$\left[\frac{500}{2}\right] = 250$

$\left[\frac{250}{2}\right] = 125$

$\left[\frac{125}{2}\right] = 62$

$\left[\frac{62}{2}\right] = 31$

$\left[\frac{31}{2}\right] = 15$

$\left[\frac{15}{2}\right] = 7$

$\left[\frac{7}{2}\right] = 3$

$\left[\frac{3}{2}\right] = 1$

994.

$\left[\frac{1000}{5}\right] = 200$

$\left[\frac{200}{5}\right] = 40$

$\left[\frac{40}{5}\right] = 8$

$\left[\frac{8}{5}\right] = 1$

249

The highest power of 5 in 1000!

$200 + 40 + 8 + 1 = 249$.

The highest power of 2 in 1000! is

$500 + 250 + 125 + 62 + 31 + 15 + 7 + 3 + 1$

$= 994$.

$$\phi(N) = N\left(1-\tfrac{1}{3}\right) = \tfrac{2}{3}.$$

$$= \cancel{729}^{\,243} \times \frac{2}{\cancel{3}_{\,1}}$$

$$= 486$$

ii) $\phi(210)$

$$\begin{array}{r|l} 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$$N = p^a \cdot q^b \cdot r^c \cdot s^d.$$

$$= 2^1\ 3^1\ 5^1\ 7^1.$$

$p = 2$, $q = 3$, $r = 5$, $s = 7$.

$$\phi(N) = N\left(1-\tfrac{1}{2}\right)\left(1-\tfrac{1}{3}\right)\left(1-\tfrac{1}{5}\right)\left(1-\tfrac{1}{7}\right).$$

$$= \cancel{210}^{\,42}\left(\frac{1}{\cancel{2}}\right)\left(\frac{\cancel{2}}{\cancel{3}}\right)\left(\frac{4}{\cancel{5}}\right)\left(\frac{\cancel{6}}{\cancel{7}}\right).$$

$$= 48.$$

iii). $\phi(600)$.

$$\begin{array}{r|l} 2 & 600 \\ \hline 2 & 300 \\ \hline 2 & 150 \\ \hline 3 & 75 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

$$N = 2^3 \cdot 3^1\ 5^2.$$

$p = 2$ $q = 3$ $r = 5$

$$\phi(N) = \left(1-\tfrac{1}{2}\right)\left(1-\tfrac{1}{3}\right)\left(1-\tfrac{1}{5}\right).(600)$$

$$= \left(\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{3}} \cdot \frac{4}{\cancel{5}}\right) \cancel{600}^{\,\cancel{200}\ 40}.$$

$$= 160.$$

Euler's function:-

The number of +ve integer less than N and prime to it called Euler function. It is denoted by $\phi(N)$

$\phi(N) = N(1-1/p)(1-1/q)(1-1/r)\ldots$

Ex.

$\phi(3) = 2.$

$\phi(6) = 2 \quad (\because \{1, 2, 3, 4, 5\}).$

$\phi(N) = N(1-1/p)(1-1/q)(1-1/r)\ldots$

Find ϕ 720.

$= p^a q^b . r^c.$

$= 2^4 . 3^2 . 5^1$

2	720
2	360
2	180
2	90
3	45
3	15
	5

$p = 2, \quad q = 3 \quad r = 5$

$\phi(N) = (1-1/2)(1-1/3)(1-1/5)$

$= 720(1/2)(2/3)(4/5).$

$= \cancel{720}^{24} . \frac{8}{\cancel{30}}$

$= 192.$

$\phi(729)$, $\phi(210)$, $\phi(600)$.

$N = p^a$

$N = 3^5$

3	729
3	243
3	81
3	27

∴ The no. of divisor excluding itself

840 = 32 − 1 = 31.

ii). 1458

2	1458
3	729
3	243
3	81
3	27
3	9
	3

$N = P^a \cdot q^b$

$= 2^1 \cdot 3^6$

$P = 2$, $q = 3$

$a = 1$ $b = 6$.

No. of divisor $= (a+1)(b+1)$

$= (1+1)(6+1)$

$= (2) \cdot (7)$

$= 14$.

∴ The no. of division excluding itself

1458 = 14 − 1 = 13.

iii). 288.

2	288
2	144
2	72
3	36
3	12
2	4
	2

$N = 2^5 \cdot 3^2$.

$p = 2$, $q = 3$.

$a = 5$ $b = 2$.

No. of divisor $= (5+1)(2+1)$.

$= 6 \cdot 3 = 18$.

∴ The no. of divisor excluding itself 18 = 18 − 1 = 17.

Find the number and sum of all the divisor 105

$$\begin{array}{r|l} 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$N = p^a q^b r^c$

$= 3^1 5^1 7^1$

$p = 3 \quad q = 5 \quad r = 7$

$a = 1 \quad b = 1 \quad c = 1.$

Number of divisor $= (a+1)(b+1)(c+1)$

$= (1+1)(1+1)(1+1)$

$= 2 \cdot 2 \cdot 2$

$= 8.$

~~Numbe~~ sum of divisor $= \dfrac{p^{a+1} - 1}{p-1} \cdot \dfrac{q^{b+1} - 1}{q-1} \cdot \dfrac{r^{c+1} - 1}{r-1}$

$= \dfrac{3^2 - 1}{3-1} \cdot \dfrac{5^2 - 1}{5-1} \cdot \dfrac{7^2 - 1}{7-1}$

$= \dfrac{\cancel{8}\,4}{\cancel{2}} \cdot \dfrac{\cancel{24}\,6}{\cancel{4}} \cdot \dfrac{48}{\cancel{6}}$

$= 192$

Divisor.

Let N be ~~a~~ number $N = ab$ then a(or) (b) is called a divisor of N it can be denoted by a divisor of N when a is not divise of N then be write $a \nmid N$ (does not divisor)

Find the no. ... excluding 1 & 480.

$N = p^a \cdot q^b \cdot r^c$.

$= 2^5 \cdot 3^1 \, 5^1$.

$p = 2 \quad q = 3 \quad r = 5$

$a = 5 \quad b = 1 \quad c = 1$.

2	480
2	240
2	120
2	60
2	30
3	15
	5

Number of divisor $= (a+1)(b+1)(c+1)$

$= (5+1)(1+1)(1+1)$

$= 6 \cdot 2 \cdot 2$

$= 24$.

$\therefore$ the number of divisor excluding 1 & 480 $= 24 - 2 = 22$.

Find the no. of divisors of 840, 1... 288. excluding the number itself.

$N = p^a q^b \cdot r^c \cdot s^d$.

$= 2^3 \cdot 3^1 \cdot 5^1 \cdot 7^1$.

2	840
2	420
3	210
2	70
5	35
	7

$p = 2 \quad q = 3 \quad r = 5 \quad s = 7$

$a = 3 \quad b = 1 \quad c = 1 \quad d = 1$.

No. of divisor $= (a+1)(b+1)(c+1)(d+1)$

$= (3+1)(1+1)(1+1)(1+1)$

$= 4 \cdot 2 \cdot 2 \cdot 2$

$= 32$.

$$= \left(\frac{1}{m+n+p+\dots}\right)^{m+n+p+\dots} m^m \cdot n^n \cdot p^p \dots$$

UNIT V

Composite Number:-

A number that has more than two factors is called a composite Number

Ex. $4 = 1 \times 4$ or 2×2

Prime number:-

A number that has exactly two number is called prime number

Ex. $3 = 1 \times 3$.

Prime to one another:-

Two numbers which have no common divisor other then one are said to be prime to one another

Ex. 13×11, 3×7.

Division of a given Number (N).

N can be expressed as the product of primes, and let N be $p^a q^b r^c \dots$ are primes.

Let 'n' be the number of divisor.

$\therefore$ The divisions of N are the terms in the expression of $(1+p+p^2+\dots+p^a)$ $(1+q+q^2+\dots+q^b)$ $(1+r+r^2+\dots+r^c)\dots$

Hence the numbers of terms in the product will be the normal of divisio

and we can easily see that the numbers of divisor is $(a+1)(b+1)(c+1)\ldots$
The divisions included one and the numbers itself

$\therefore$ The sum of divisions

$$S = \frac{p^{a+1}-1}{p-1} \frac{q^{b+1}-1}{q-1}$$

$$\frac{r^{c+1}-1}{r-1} \ldots$$

Find the number and sum of all the divisors 360

$$N = p^{a} q^{b} r^{c}$$

$$= 2^{3} 3^{2} 5^{1}$$

2	360
2	180
2	90
5	45
3	9
	3

$p = 2 \quad q = 3 \quad r = 5$

$a = 3 \quad b = 2 \quad c = 1.$

Number of divisors $= (a+1)(b+1)(c+1)$

$= (3+1)(2+1)(1+1)$

$= 4 \cdot 3 \cdot 2$

$= 4(3) \cdot 2$

$= 24.$

Sum of divisors $= \frac{p^{a+1}-1}{p-1} \cdot \frac{q^{b+1}-1}{q-1} \cdot \frac{r^{c+1}-1}{r-1}.$

$= \frac{2^{4}-1}{2-1} \cdot \frac{3^{3}-1}{3-1} \cdot \frac{5^{2}-1}{5-1}$

$= 15 \cdot \frac{\cancel{26}^{13}}{\cancel{2}} \cdot \frac{24}{4}$

Replacing ~~a~~ ~~by~~ a by $a-1, a-2, \ldots 3, 2,$

$$(a-1+1)^p \equiv [(a-1)^p + 1] \ [\text{mod } p]$$

$$a^p \equiv [(a-1)^p+1] \text{ mod } p \longrightarrow (1).$$

$$(a-2+1)^p \equiv [(a-2)^p + 1] \text{ mod } p.$$

$$(a-1)^p \equiv [(a-2)^p + 1] \text{ mod } p \longrightarrow (2)$$

$$(a-2)^p \equiv [(a-3)^p + 1] \text{ mod } p \longrightarrow (3)$$

$$\vdots$$

$$4^p \equiv (3^p+1) \text{ mod } p \longrightarrow$$

$$3^p \equiv (2^p+1) \text{ mod } p \longrightarrow (a-2)$$

$$2^p \equiv (1^p+1) \text{ mod } p \rightarrow (a-1)$$

adding eq^n $1, 2, 3, \ldots \ldots (a-2) \ (a-1)$ we get

$$a^p + \cancel{(a-1)}^p + \cancel{(a-2)}^p + \ldots\ldots + 4^p + 3^p + 2^p$$

$$\equiv \cancel{(a-1)}^p + \cancel{(a-2)}^p + (a-3)^p + \ldots\ldots + 3^p + 2^p + 1^p$$

$$+ (a-1) \text{ mod } p.$$

$(a-1)$ times

$$a^p \equiv (1^p + (a-1)) \text{ mod } p$$

$$= [\cancel{1} + (a-\cancel{1})] \text{ mod } p.$$

$$a^p = a \text{ mod } p.$$

$$a^p - a = 0 \text{ mod } p.$$

$$a[a^{p-1} - 1] \equiv 0 \text{ mod } p.$$

Since a is prime to p, $a^{p-1} - 1$ is divisible by p.