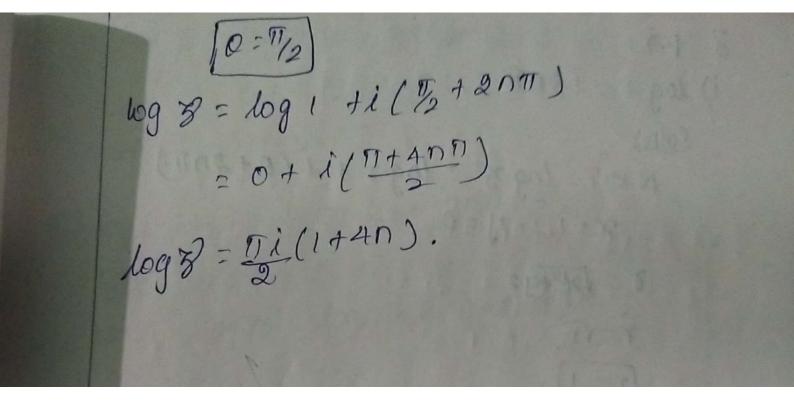
UNIT-V Logarithm of complex numbers Def: let z = r (coso + i sin a) be a non-zero complex numbers we define logy = logr + i o Loxaly xircoso; y=rono 7=8(050+isin0) e io : koso ti kino Z-8. eio. Taking log on both sides log Z = log (reio) = logr + loge 10 logz = log r + io MOTE: If we denote general value of log I bo by log I. i) then log 8= log \$ + i 2 n 11 , n & \$ log 3 = logy + io + ion T log 8 = log T + ilotanT). ii) z= x+iy xtiy=r(cosot étina) (x+iy)= 181 / coso+ isin 0/ 122tp = 18/1 (8244) = Y = 0 = tan (1/2) log & = log (2+y" + i [tan" (4) + anti]

```
Theonem:
  69 (3,82) = log 8, + log 82
  Awoof !
    Z = r (coso+irsno)
    B,=8, (wso,+issno,)
    82 = 80 (cos 00 + irsin 00)
 B, Ba = r, (coso, + i sin o,) · r, (coso, + isino)
       -8, 72 (coso, cosoa + i coso, sinoa + issino,
                            6508 - 18100, sino2)
       = r, r, [(coso, coso, -sino, sino,) +
                    i(coso, trino, + (sino, cos 02))
       = 81 82 [(cos (0, +02)) + i sin (0, +02)]
8, H2 = 21 ×2 [e 10,+02)]
   Taking log on both stoles.
log (3,32) = log (8,82). log [e i(0,+02)]
log(8,80) = logr, +logr2 + i(0,+02)
            = log8, + log8, + i0, + i0,
      = logr, + logr2 + i (0, + 2nn) + i (0, + 2nn)
            -[log91,+i(0,+2n11)]+[log82+i(0)+2n5)
log (7,82) = log 3, + log 82.
```

```
1. Find log (112)
   N.KT: log & = log Tti (0+anT).
   r: 1/2° +43
   0 = tan-1 (4/21)
   7:111.
       F: (2)
      0=tan-1(1/1)
       10 = 7/4
   log 8 = log (va) + i (= +2nTI)
        = log v2 + i [TT+8 n TT]
        = logva + IIi (1+8n).
2. Find log (i)
   Soln:
     N.K.T: 609 8 = log r + i (0+20 TT)
       x=0, y=1
    8 = V2+42
    KELL [LE]
    0 = tan-1 (3/x)
       = fan (1/0)
       = tan-1(2)
```



```
3. P.T:
 i) log 1 = i & n TT
     N.K.T: Log 8 = Log 8 + i (0+2011)
   Soln:
       X=1, 4=0
     8 = 12/0 ty2
      rovi
      (r=1)
      0=tan=1 (%)
       0 = tan-1(0)
       10=0
  log 8 = log 1 + i ( = + 2 n T)
         =Otian T
   log & = ian TI
ii) log-1 = i (2n+1) TT
   N.K.T: 109 8: log + i(0+2n1)
         x=-1, y=0
         7: 1x2+42
         roll
          8:21
          0 = tan (9/x)
            = tan-1 (%)
                               if tan-1(-1)
                                 the Value of 0:7
            = tan-100)
         0=0
```

```
log 3 = log 1 + i (#+2n11)
           = 0+ i(2n+1)11
   log 8 . - 1 (2N+1) 11
iii) log (-1+i) = - log 2+i(2111+31/4)
   Soln:
   W.K.T: log 8 = log & +i(0+2n71).
         2=-1, 4=1
         7= 1+1+1
                                  V) log Vi = i(40+1)
      8=12
         0 ; tan -1 (/21) vi) log (-1-i) -
           0 = tan-(-1)
           0=311/4
  log 8 = log (2) 12 + i (3II + 2n II)
         -12/6 g& + il 3 17 + 8 P. T = - 6 10 g2 + i (2 nTi + 3/1)
(v) log (-1) = i(An+U)/2
  Soln:
  N.K.T: log 8 = log 8 + i (O+2nTI)
          2=0,4=-1
          8 = \x2 +42
          8:11
         [r=1]
          0-tan-1(9/2c)
           = tan-1 (-1/6) = tan-160)
```

$$log \ 8 = log \ 1 + i \left(\frac{11}{3} + 2n\pi \right)$$

$$= 0 + i \left(\frac{11}{3} + 2n\pi \right)$$

$$= i \left(4n + 1 \right) \frac{1}{2}$$

$$x = 0, \ y = \sqrt{1} = 1$$

$$0 = tan^{-1} \left(\frac{1}{2} \right)$$

$$0 = tan^{-1} \left(\frac{1}{2} \right)$$

$$= 0 + i \left(\frac{11}{3} + 2n\pi \right)$$

$$= 0 + i \left(\frac{11}{3} + 2n\pi \right)$$

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$$= 0 + i \left(\frac{11}{3} + 2n\pi \right)$$

$$= 1 \left(4n + 1 \right) \frac{1}{2}$$

$$= 1 \left(2n + 1 \right) \frac{1}{2}$$

```
Def: If I ov are any two complex numbers then
   we define,
          Zw = e 269%.
   Note: since log of has infinite number of values
   for I. I also has infinite number of values.
1. a) If i atib = atib. P.T a2+62 = e-(4n+1) 116
  (Or)

DIF ixtiy = AtiB P.T A2+B2 = e-(+N+1)TTY.
  a) Soln:
   an, iatib = atib
          Z=E, Q=a1ib
   NKT: ZO = ewbg 3
          iatib = e latib) logi >0
           0 = tan-1 (4/x)
          = tan-1(1/0)
             = tan-1 (20)
          0 = 1/2
   log 7 = log 7 + [(0+211)
   logi = log 1 + [( 1/2 + 2 n TI)
   logi = i (THANTI)
   logicity (Anti) -> @
```

Sub @ in O (An+1)

[aib =
$$e^{(aib)}i\eta_{b}(An+1)$$

[aib = $e^{(aib)}i\eta_{b}(An+1)$

[aib = $e^{(a-b)}\eta_{b}(An+1)$

[aib = $e^{(a-b)}\eta_{b}(An+1)$

[aib = $e^{(a-b)}\eta_{b}(An+1)$

[aib = $e^{(a-b)}\eta_{b}(An+1)$

[aib = $e^{(ab)}e^{(an+1)}$

[aib = $e^{$

$$= \frac{3i+1}{3+i} \times \frac{3-i}{3-i}$$

$$= \frac{9i+9+8-i}{(8)^2-(1)^2}$$

$$= \frac{8i+6}{9+1}$$

$$= \frac{9(4i+3)}{9+1}$$

$$= \frac{4}{5}i+\frac{3}{5}i$$

$$= \sqrt{(\frac{3}{5})^2+(\frac{1}{5})^2}$$

$$= \sqrt{(\frac{3}{5})^2+(\frac{3})^2}$$

$$= \sqrt{(\frac{3}{5})^2+(\frac{3}{5})^2}$$

$$= \sqrt{(\frac{3}{5})^2+(\frac{3})^2+(\frac{3}{5})^2}$$

3. P.T: 69 20 = W692 Soln' Land ware any two complex number : YW = oblog x Taking log on both tides. log x is = wlog x. Summation of series: Some of the important methods of summation are i) Difference method ii) Angles in with metics progression melhod. iii) brugory method. Difference method: In this method we ase various trigonometric formula of split up each term unit difference of two terms such that one term of each term appears as one of the native term of the next term and summation gives the required nexitts. Aroblem: sino sinao + sinao sinao 1. Sum the series: (0r)

```
lose o lose 20 + cose 20 cose 30+ ... + n terms
Soln
let Sn = sinokingo + singokingo + ... + nter ma
Here the nik terms In = sinnomin(n+130
   now x & - by Min 0
In = I sino sincontido
    now adding & sub (no)
In = sino (ot no -no) 7

Sino Sino Sin(n+1)0
    = sino sinoxin(n+1)0
    =\frac{1}{\sin \theta} \left[ \frac{3\sin(n+1)\theta \cos n\theta - \cos(n+1)\theta}{\sin n\theta \sin(n+1)\theta} \right]
    = 1 Sin(n+1) ocos no - ws(n+1) o sinno sin(n+1) o sinno sin(n+1) o sinno sin(n+1) o
n=2, Tn= sina [606 0 - cot(n+1)0]
T2 = coseco [ Lot 20 - Lot 30]
T3 = lose (0 [ast 30 - cot 40]
Tn-1= cose co [cot (n-1) 0 - cot no]
In = losec o [cot no - cot (n+1) o]
 Sn = T, +T2 + T3 + ... + Tn
     = cose c c [ cot 0 - cot (n+1)0)
```

2 Sum of the series Seco sec 20 + sec 20 sec 30 + .. + n terms Soln: Sn = Sec 0 sec 20 + sec 20 sec 30 1 ... + n texm Sn = 1 cos 20 cos 20 + Here. The nik terms. Tn = (05 n 0 cos (n+1) 0 Now, x & - by kin o Th = Sing wish o cos (n+1)0 Now adding & sub no = 1 [sin (0+n0-n0)]

sino [cos no ws (n+1) co] Sino [con cos contido] = 1 Sin(n+100cosho - Sinnocos(n+1)o Sino Losno los (n+1)0 - Log Sincat Down ob - wolln+128 sinng = 1 (sin(n+1) ocusno cos(n+1) to sinno cos(n+1) to sinno cos(n+1) to

```
= 1 Sin(n+1)0 - sin no ]

Sin o Los (n+1)0 - Cosno
    Th = coseco [tan(n+1)0 - tan no]
    put
    n=1, tanka serendari sama
    T, = cosec o (tan 20 - tan o)
   n=2,
   Ta = cose( o (tan 30 - tan 20)
   n=3,
   T3 = cuse (0 (tan 40 - tan 30)
   In = losec o [tan (n+1)0 - tan no]
   Sn = T, + T2 + ...
   Sn = coseco [tan(n+1) o - tano].
3. S_n = \frac{1}{\cos 0 + \cos 30} + \frac{1}{\cos 0 + \cos 50} + \frac{1}{\cos 0 + \cos 70} + \dots n \text{ terms}
  Soln!
Here In = (050 + cos (2n+1)0
  N.K.T: COS (A+B) + COS (A-B) = QLOS A COS B
           sub A = (n+1)0, B=n0
        Cos (n+1+n)0+cos (n+1-n)0 = 2 cos (n+1)0 cos no
       (05 (2n+1) 0 + coso = 2 (05 (n+1) 0 cos no.
          210 (n+1)0 arsno
```

```
Now x & - by son o

To sino [cos (n+1) o cos no)

- Resec o [cos (n+1) o cos no)

- Resec o [cos (n+1) o cos no)
     =2005e1 0 [05(n+1)0 ws no]
     = 2 cosec o [sin(n+1) 0 cosno - cos(n+1) 0 sinno
cos(n+1) 0 cosno
      = 2 wser o sin(n+1) o ws no ws(n+1) o winno ws no ws(n+1) o ws no
      = 2 wseco [sin(n+1)0 _ sin no]
  Tr = 2 wseco [tan(n+1)0 - tan no]
  Put,
   n= 1
   T, = 2 wseco [tan 20 - tan 0]
   n=2,
   T2 = 2 wseco (tan 30 - tan 20).
   Tz = 2 coseco (tan 40 - tan 30).
   Tn = 2 wseco [tan[n+1) o - tan no]
    Sn = TitTat ...
   Sh - 200sec o Stan (n+1)0 - tanno
```

 $T_n = \frac{1}{2} \left[\frac{1}{2} \tan(3^n o) - \tan(3^{n-1} o) \right]$ $S_n = \frac{1}{2} \left[\frac{1}{2} \tan(3^n o) - \tan(3^n o) \right].$ 6. Sn = Sin x cos2x - cos2x sing 2 sinax cos4x Put x = 7 - 0 Sn = - (=-0) cosa(=-0) cosa(=-0) sin 3(=-0) Sirt(3-0) LOA(1/6-0) Cos o cos 20 Cos 20 Cos 30 Cos = - [loso cos 20 + cos 20 cos 40 + cos 30 cos 40 Th = - cosno cos (n+1)0 x & + by kina Add & soub by no sino [sin(o+no-no)] = - I [sin [(n+1)0-no]]
sino [cos ho cos(n+1)0] - 1 Sincht) Owsho - coscht) o sin no sino L cosno coscht) o Cos (n+1)0 - sin no

```
= = [tan(n+1)0 - tan(no)]
  In =- coseco [tan(n+1)0-tanno]
  hal,
    Ty=-cose ( o [tan 20 - tan o]
   1=2
    Tg = - coseco [tan 30 - tan 20]
    T3 = - cose co [tan 40 - tan 30]
T: 1-1
     This = lose co [tan no - tan (n-1)0]
   n=n
     Th:-coseco[tan(n+1)0-tanno]
  :. Sn = - cose co [tan(n+1)0 - tano].
7. Find Sn = tan-1(/3) + tan-1(/4) + tan-1(/13)+...n texm
  Sn: tan-1[1/1(2)] + tan-1[1/2(3)] + tan-1[1/3(4)]
  In=tan- / [+n(n+1)]
          Add & sub by n.
     = tan 1 (1+h-h)
  = \tan^{-1} \left[ \frac{(n+1)-h}{1+n(n+1)} \right]
  ftan-A-tan-B-tan-1[A-B-74
   Th = tan (n+w - tan (h)
```

```
T_1 = \tan^{-1}(2) - \tan^{-1}(1)
                             Kalpana
  To = tan (3) - tan (2)
 17:3
   T3 = tan (4) - tan (3)
   Tn-1=tan-1 n- tan-1.(n-1)
 n= n,
   Tn: tan-1 (n+1) - tan-1(n)
 :. Sn = tan-(n+1)-tan-(1).
8. Find Sn = tan [1 1 + 1 + 12] + tan [1+2+2] + tan [1+3+32] + ... n/en.
    S_n = tan^{-1} \left[ \frac{1}{1+n+n^2} \right]
      =tan-[i+n(n+i)]
        Add & sould by n.
      -tan-1 [1+n-n]
      :tan-1 [(A+1)-n]
  gtan-1A-tan-1B=tan-[1-AB]
  In = tan-1(n+1) - tan-1 n
    Ti = tan-1(a) - tan-1(1)
```

```
n:2,
     To: tan-1(s) - tan-1(a)
   h-3.
      Ta = tan-1(4) - tan-1(8)
   h - i
   n=h-1,
      The 1 = (an -(n) - tan - (n-1)
   h-h,
      Th = tan (n+1) - tan (n)
    : Sn = tan- (n+1) - tan- (1).
5. Find the sn=tanosec20+tan20sec40+
                              tanApsec80+. man
    Sn = Sin a + Sin 2 a + Sin 2 a + Sin 4 a ws 4 rows 80 for + norm
    T_n = \frac{\sin(2n-1)\alpha}{\cos(2n-1)\alpha}
         = sin(2n-1)0
(05(2n-10)cos 2n0
     = 4 x by 2 cos (2n-10)
     - 2001 de 2005 (2n-10) cos (2n-10)
     - fsin (2x2n-18)
- aus (2n-10) (05(2n-10) (052n0)
     = 1 sector 36 [(2n 0) - (2n-10)]

cos (2n-10) cos 2n0
```

= 1 secon 3 sin 2 0 cos 2 0 0 cos 2 0 0 sin 2 0 - 107 cos(2n-10) cos(2n0) =\frac{1}{2}\sec(2^n-\frac{1}{2}\oldowno\frac{\sec(2^n-\frac{1}{2}\oldowno)}\frac{\sec(2^n-\frac{1}{2}\oldowno)}{\cos(2^n-\frac{1}{2}\oldowno)\cos(2^n\oldowno)}\frac{\cos(2^n-\frac{1}{2}\oldowno)\cos(2^n\oldowno)}{\cos(2^n-\frac{1}{2}\oldowno)\cos(2^n\oldowno)} $=\frac{1}{2}se(2^{n-1})o\left[\frac{s(n 2^{n} 0)}{cos 2^{n} o} - \frac{s(n 2^{n-1} 0)}{cos 2^{n-1} o}\right]$ Th= 1 sec (2) oftangno - tan 2 no o n=1, = sec(2^-)0

T, = [fan 20 - tan 0] n=2, sec(29-1)0 To= [fan 40 - tan 20] n=3 /2 sec(2n+1)0 T3 = (Fan 80 - tan 40) h = n - 2, seconino $T_{h-2} = \frac{1}{2} \left[\frac{1}{4} a n^{2} a - \frac{1}{2} a n^{2} a \right]$ n=n-1, = secon 10 - tan 2 n-2 a)

Th-1 = [tan 2 n-10 - tan 2 n-2 a] h=h, sectorio no - tan 2n-10)

Th = Ean 2no - tan 0

Sh = Fan 2no - tan 0

```
9. Find Sn = tanx tan (x+y) + tan (x+y) tan (x+ >4
             +tan(x+2y)tan(x+3y)+... nterm
  Solo:
    Sn = tanx tan(x+y) + tan(x+y) tan (x+2y)
                                t...ntrms
   Th = tan [x+ (n-1)y] tan (x+ ny).
   let tan y= tan [(x+ny) - (x+(n-1)4)]
   tan (A-B): tan A - tan B
                1+ tangtan B
   :. tany = tan(x+ny) - tan[x+(n-1)y]

(+ tan(x+ny)tan[x+(n-1)y]
   1+ tan(x+ny) tan[x+(n-1)y] = 1 [tan(x+ny)-tan(x+cn-n)]
   1+Tn = coty [tan (x+ny) - tan (x+(n-1)y)]
   Tn = wt y [tan(x+ny) - tan(x+(n-1)4)] -1
     To: coty (tan (xty) - tan x] -1
   n=2,
      To=(oty [tan(x+24) - tan(x+4)]-1
      To = lot y [tan (x+3y) - tan (x+2y)]-1
   h=3,
   n=n-1
       T3 = cot y [tan (x+(n-vy) - tan (24(n-2)4)]-1
```

non, In = loty [tan (x+ny) - tan (x+(n-1)4)]-1 isn = wty [tan (x+ny) - tanx] -1 10. Find Sn - Sin 20 + Sln 10 + Sin 60 + norm
COS O COS 30 4 505 30 COS 50 COS 50 COS 50 $S_n = \frac{\sin 20}{\cos 2\cos 20} + \frac{\sin 40}{\cos 20\cos 50} + \dots$ nterms Tn = sin 2n 0 cos(2n+1)0 X & - by 2 coss (sin 0 = dsino [asin 2no. sin a]. $=\frac{1}{2}\cos(0)\left[\frac{\cos(2n-1)\cos(2n+1)0}{\cos(2n-1)\cos\cos(2n+1)0}\right]$ Th = cosec a [sec (2n-1)0-sec (2n-1)0]. T, = cosec 0 [sec 30-sec 0] 12 = wser o [xec 50 - sec 30] 73 = cosec o [sec 70 - sec 50].

Tn-1 - cose co [dec (2n-1)0 - 18(c (2n-2)0) Tn = Loseco [sec (2n+1)0 - sec (2n-1)0] In = cosec o [sec (2n+1)0 - sec o] 11. Find sn = sin & sin Soln: sin 20 - sin 40 + ... nterms Sn = sin 0 16 in 30 - sin 30 16 in 50 Tn=H) "sin 2n 0
Sin (2n-1) 0 /sin (2n+1) 0 x 4 ; 2 000 0 $= \frac{(-1)^{n+1}}{2 \cos \left[\frac{2 \sin (2n-1)0}{\sin (2n-1)0} \frac{\cos (2n+1)0}{\sin (2n+1)0}\right]}$ = $\frac{(-1)^{n+1}}{2}$ seco [$\frac{\sin(2n+1)a}{\sin(2n-1)a}$ sin(2n+1)a $\frac{\sin(2n-1)a}{\sin(2n-1)a}$ = (-1) 1/8(0 (Sin(2n-1)0 min(2n+1)0 sin(2n-1)0 min(2n+1)0) Th = [200 sec (2n-1) 0 + cos (2n+1)0] T, = CHOSA SELO [cose co à cosec 30] To - CLYSTATISCO PLOSEL 30 7 COSEC 50]

```
To = Seco [cosec 50 - cosec 70]
  h=n-1,
  Th-1 = seco [cosec (2n+1)0 - cosec (2n-100)
   Tn = seco [cosec (2n-1)0 + - cosec (2n+1)0]
  :: Sn = (-1) n+ 1 seco [cosec 0 + cosec (2 n+ 1) 0].
  brugory method:
1. Theorem: If - IT = 0 = IT
         then o = tan o - = tan 30 + + tan 50 -
  Proof:
   consider 1+itano
  litano: 1+ i sino coso
          = COSO + LISINO
COSO
           = sec o (coso + I sino)
  11 itano = seco.eio -> 0
  Taking log on both rides
  log (Hitano) = log (seco.eio)
                = log (seco) + log eio
  log (Hitano) = log (seco) +io -> 0
    Since, - 11 = 0 = 17 => 1 tan 0 | = 1
```

tan(=) = tano = tan(=) $-1 \leq tano \leq 1$: log(14 itano) is expanded in logorithmic log (1+i tan 0) = (itan 0) - (itan 0) + (itan 0) series. - (itano) 4 (itano) Now sub log(1+itan 0) = log(seco)+lo log(seco)+ io: Itan o+ + tan o - itan o - tanto titanso -... Equating imaginary parts 0 = tano - tanso + + tanso - 2. Conolloug: If -1= 2 = 1 then tan-1(21)= x-323++x Proof: put a = tano in above thrm The condition of the theorem is, 一十丁二〇三寸 is can be written as ton(==) = tano = tan //4 -1 = tan 0 = 1ta Hence, $-1 \leq x \leq 1$ If tano= 2 => 0 = tan 2 0: tano - 1 tanso + + tanso - -...

 $tan^{-1}(x) = x - \frac{1}{3}x^{2}t - \frac{1}{5}x^{3} - \cdots$ Note: put 2 = 1. 事=1-まま・ナーナナ・・・ 3. The general form of breegory's theorem: Theorem: If NTI- + TT < 0 < NTI + + TT Then, 0-nTT = tano- = tan30+ = tan50. Proof: 1 184 Let 0 = nT + \$\phi\$ where, => \$\phi = 0 - nT| - 上丁 = ゆと 上丁 Consider, Hitano = (+itan (nTT+p) · Hitain p - 1ti mn p cosp = cosø + i sinø.

cosø = secp (cosotisting) 1+itano = sec \p . eip Taking log on both sides. log (1+itan 0) = log(sece) + ip Since, $-4\pi \leq p \leq 4\pi \Rightarrow |tanpl=1$ tan(==) = tan = tan = -1 = tan 0 = 1 .. log (1+itano) is expanded in log arithmic series.

$$S_{x} = \tan^{-1} \left(\frac{2(\frac{1}{3})}{1 - (\frac{1}{3})} \right)^{2} + \tan^{-1} \left(\frac{2x}{1 - x^{2}} \right)$$

$$= \tan^{-1} \left(\frac{2}{3} \times \frac{9}{8} \right) + \tan^{-1} \left(\frac{1}{4} \right)$$

$$= \tan^{-1} \left(\frac{3}{4} \times \frac{9}{8} \right) + \tan^{-1} \left(\frac{1}{4} \right)$$

$$= \tan^{-1} \left(\frac{3}{4} + \frac{1}{4} + \frac{1}{4} \right)$$

$$= \tan^{-1} \left(\frac{3}{4} + \frac{1}{4} + \frac{1$$

$$S_{1} = \frac{\pi}{4} - tan^{-1}(\sqrt{3})$$

$$= \frac{3\pi}{4} - \frac{\pi}{6}$$

$$= \frac{3\pi}{4} - 2\pi$$

$$S_{1} = \frac{3\pi}{12}$$

$$S_{1} = \frac{3\pi}{12}$$

$$S_{2} = \frac{3\pi}{12}$$

$$S_{3} = \frac{3\pi}{12}$$

$$S_{4} = \frac{3\pi}{12} + \frac{1}{5} \frac{1}{3^{2}} - \frac{1}{7} \cdot \frac{1}{3^{3}} + \cdots$$

$$S_{5} = \frac{2\pi}{3} \cdot \frac{1}{3^{2}} + \frac{1}{5} \cdot \frac{1}{3^{2}} - \frac{1}{7} \cdot \frac{1}{3^{3}} + \cdots$$

$$S_{6} = \frac{2\pi}{3} \cdot \frac{1}{3^{2}} + \frac{1}{5} \cdot \frac{1}{3^{2}} - \frac{1}{7} \cdot \frac{1}{3^{3}} + \cdots$$

$$= \frac{1}{5} \cdot \frac{1}{73} - \frac{1}{3^{2}} \cdot \frac{1}{73} \cdot \frac{1}{73} + \frac{1}{73} \cdot \frac{1}{3^{3}} + \cdots$$

$$= \frac{1}{5} \cdot \frac{1}{73} - \frac{1}{3^{2}} \cdot \frac{1}{73} \cdot \frac{1}{$$

If
$$|\alpha| < 2 - 1$$
 using gregory's social.

PT: $R \left[2 - \frac{2^3}{3} + \frac{x^5}{5} - ... \right] = \frac{8^3 \times 2}{1 - x^2} - \frac{1}{3} \left[\frac{2^3 \times 2}{1 - x^2} \right] + \frac{1}{5} \left[\frac{2^3 \times 2}{1 -$

: These two inequalities are true only for, 4 12/E 12-1 Angles in Arithmetic progression: To find the sour of the sines of a socies of angles which are in A.P. Soln: Let the series be Sn = Sind + 18in (d+B) + 18in (d+2B)+ ...+ sin (2+(n-1)B). Multiplying both sides by a sin(1/2), 2 sin (1/2) Sn = 2 Sin 2 Sin (1/2) + 2 sin (d+1/2) sin (%) + - + 2 kin (x+(n-1) 8) sin (B). W.K.T: & sin A sinB = MACA-B)- MACA+B) 2 sin (8/2) Sn = (015(d-8/2) - cos (d+8/2)+ (cos(d+B-B/)- cos(d+B+B/) [cos(d+(n-1)B-B/2)- cos(d+(n-1)B+B) = [ws(d-1)-cos(d+1/2)+ [cos (2 - P/2) - cos (2+ 3B)]+.. [cos(d+(n-3)B)-605(d+(n-1)B) = cos(2-B)-cos(2+(n-1)B)

```
2 sin ( 1/2) sh = 2 sin[d + (n-1) ] sin(n)
        = 20x Part x 1 x 8/2 1/2/3
    Sin(1/2) Sh = Sin(2+(n-1) P/2 ] rain (nP/2)
   Sn = Sin [d+(n-1) B/2] 18in (n B/2)
                   Sin (B/2)
2. To find the sum of the series wrines of a
   sories of angles which are in A.P.
     Let the series be
   Soln:
   Sn= los d + los (d+β) + los (d+2β) + ...+ los (d+(n-1)β)
   x by 2 sin( 1/2) on both sides
  a\sin(\frac{\beta}{2})s_n = a\cos a \sin(\frac{\beta}{2}) + a\cos(a+\frac{\alpha}{2})\sin(\frac{\beta}{2})
              +2\cos(\alpha+2\beta)\sin(\beta/2)+\cdots+
        2 cos (d+(n-1)B) kin (B/2).
    W.K.T: 2 cos A sinB = sin (A+B) - sin(A-B).
  2 sin(P/2) sn = sin(d+P/2) - sin(d-P/2) +
                   sin(2+03+8/2)-sin(2+08-8/2)+
    sin(d+2\beta+\beta/2)-6in(d+2\beta-\beta/2)
     1-1- Sin (d+(n-1)B+B/2)-
                             Sin [a+(n-1) B-P/2]
```

= and + P/2) - 1810 (2-P/2)+ sin(2+38)-18in(2+P/2)+ Sin(d+5B)-18in(d+3/2)+...+ Sin(d+(n-シ)β)ーらか(d+(n-シ)β) = - kin (x-P/2)+kin [a+(n-1)] = 1sin [x+(n-12)] - sin (x-1/2) 25in B/2 Sn = 2 cos (nB/2) cos [d+(n-1)B] Sin By Sn = WS (nB) cos (2 + (n-1) B) Sn = Gir (nB) cos (2+ (n-1) B) $\sin \beta/2$ 3. Find Sn = Sin & + sin 2x + sin 3x+...nterms Sn = Sin & + kin 2 x + kin 3 x + ... n terms. Solo: x by & sin(1/2) on both sides 2 sin (4/2) Sn = 2 sin & sin (4/2) +2/5 n 2 x sin (4/2) + 2 sin 32 sin (4/2) + ...+ asinna sin(4/2) WO. KiT: 2 sin A sin B = WS (A & B) - COSCA+B) 2 Sin (d/2) Sn = cos (d-d/2) - cos (d+d/2)+ ws(2x-d/2)- ws (2x+d/2)+ 605 (3d-d/2) - 605 (3x f %)+. Cos(nd-9/2)-605(nx+9/3)

```
= (05(42) - cos (3d) + cos (8d/2)
                    - los (5d/2) + cos (5d/2)
                      los (6d/2) + ... + los (2nd-d)
                                    - los (2nx+d)
                 = (05 (d/2) - cos/2nx+x)
     = \cos(\frac{4}{2}) - \cos(\frac{2n+1}{4})
  asin d/2. Sn = asin (n+1) d/2. sin (nd/2)
                 · Sin (n+1) 4/2 . sin (n4/2)
                           Sin d/2
4. P.T: tan nd = sind+ ling x+ ling x+ ling terms
                  cos d+ cos 3d+ cos 5d+...n terms
   Sind+ sin(d+B)+sin(d+2B)+.. = sin(d+(n-UP/2)
  Soln:
  W.K.T: P. H.S:
                                         Sin (8/2) -0
 (05 d + (05 (d+p) + ws(d+2p)+ = Eos(d+(n-v) P/2) sin (nP/2)

Sin(p) 1
 Sin 27 Min 32 + Min 52+ .. = Sin 2+ Min (2+22)+
                                  18in (2+ A 2) + ...
          d=d, B= 2d subin O
0=> = sin [x+(n-1) 8x] sin ( 2x)
                     Kin (22)
```

& Sin(d+nd-d) sin(nd) voin a = sin (nx) sin (nx) sin 2 $=\frac{\sin^2(n\alpha)}{\sin\alpha} \rightarrow 3$ fin, los & + los 3 & + los 5 & + ... = los d + los (& + 2 x) + cos (x+4x) +... d=d, B= &d youlb in O (D=) = cos [d+(n-1) 2] . Sin (n 2 x) sin (2x) - Ws (d+nd-d) sin(nd) = los (na)sin(na) -> @ Sin 2 Jan 19-15 mis 4 son 3 - A Sin2 (nd) sind (sin(nd) = sin(n2) los (nx) = tan(nx) - BLIBS

5- Find Sn = 6052x + cos (x+B) + cos2 (x+2B)++n terms Hence find cos2x+ cos22x+ cos23x+ ... + nterms Sn = cos2 + cos2 (2+B) + cos2 (2+2B)+..+n terms = [1+ 6052x] + [1+ 6052(x+B)] + [1+ 6052(x+2B] do in terms = [1 + 1 + ... + n terms] + 1 [los 2d + los 2 (d+p) + 650(2+3B)+..+ = $\frac{n}{2} + \frac{1}{2} \left[\cos \left(2 \times + (n-1) \frac{2\beta}{2} \right) \right] \sin \left(\frac{n + \beta}{2} \right)$ = \frac{h}{2} + \frac{1}{2} \left[cos [2x + (n-1)B] sin (nB)]

Sin B (ii) cos2 + cos2(d+B)+ cos2(d+2B)+-+nterms = 2+ = [ws[2x+(n-1)B]sin mB] Sin B put p=2 in egn O 6052 + 6052 22 + 6052 32 + .. +n terms $=\frac{n}{2}+\frac{1}{2}\left[\cos\left(2\alpha+n\alpha-\alpha\right)\sin\alpha\alpha\right]$ = 1 + 1 [ws (2+n2) (sin n2) Sin2

Find in = 605 d + 605 22 + 605 3 d + . + 1 teom Solo: Sn= + [cos 3 x + 3 cos 2] + + [cos 6 x + 3 cos 2 x] + 1 [cos 92 + 3 cos 32] +... + n tem = 1 [cos 3d + cos 6d + cos 9d + ... + n berms] + 3 [cos d + cos 2 d + cos 3 d + ... + n lems] 2=3d, B=3d = 4 [80s [3x+(n-1)3x] sin [n3x] sin (32) 十号 [105 2 + (n-1) が (6) [型] $=\frac{1}{4}\left[\cos(3\alpha+3n\alpha-3\alpha)\sin(\frac{3n\alpha}{2})\right]$ $\sin(3\alpha/2)$ +3 [aus (2+n2-2) 15in (m2)] = 1 [ws(3x+3nx) /sin (3nx)] +3 [ws (d+nd) toin (nd/2)] Sin (12/2) = 4 [cos[(n+1)3x] sin 3nx] +3 [Ros(1) 2] (sin (1)2) Scanned with CamScanner

7 Find Sn = Cos & cos 3x + cos 3x60s 5x + los 5x 60072 +..+n terms solp " Sn = 2 /2605 x 6053 x + 26053 x 6055 x + 2605 5 x 60572 WKF 2005A COSB = COSCA+B)+ COSCA-B) Sn = = [ws 4 x + cos 2 i + ws 8x + cos 2x) + (cos 12x) + cos 2d)+ ··· +n terms $=\frac{1}{2}\left[\frac{\sin(h_{1})}{\cos(4\alpha+(h-1))}\frac{4\alpha}{2}\right]\sin(4n\alpha)$ $=\frac{1}{2}\left[\frac{\sin(h_{1})}{\sin(h_{2})}\frac{\sin(4n\alpha)}{\sin(h_{1})}\frac{\sin(4n\alpha)}{\sin(h_{2})}\frac{\sin(4n\alpha)}{\sin(h_{1})}\frac{\sin(4n\alpha)}{\sin(h_{2})}\frac{\sin(4n\alpha)}{\sin(h_{1})}\frac{\sin(4n\alpha)}{\sin(h_{2})}\frac{\sin(4n\alpha)}{\sin(h_{1})}\frac{\sin(4n\alpha)}{\sin(h_{2})}\frac{\sin(h_{2})}{\sin(h_{2})}\frac$ Sin (4x) + (1/6522)} = 1/2 [BS(4x+(n-1)2x) sin (2nx)]+(ncos2x)}
Sin 2x $=\frac{1}{2}\left[\frac{(\omega s)(2\alpha+2n\alpha)}{\sin(2n\alpha)}\right]+(n(\omega s)(2\alpha))^{2}$ = 1 f cos (n+1) 22 sin(2nx) Je(n cos 2x)} $=\frac{1}{2}\left\{\left[\frac{2\cos(n+1)}{\sin(2n+1)}\right]+(\ln(\cos(2n+1))^{2}+(\ln(\cos(2n+1))^{2}+(\ln(\cos(2n+1))^{2})\right\}$ 190 9 19. 100 6 0 1 1 1/2 AIS

s and in sind - win 2d + win 3d - win 12+ Saln W.K.T: Sin(17+22) = - 1811 2d Sm (211+32) = 18in 3 d sin (311+42) =-18in 42 Sin (411+5d) = 18in 5d sin (517+62) = - 600 6d. By using the above, Sn = Sind + (sin (TT+2) + (sin 30 + (sin (3) +40)+. = Sin.d + 65n [d + (d+11)] + 65n [d+2(d+11)] + Sin[d+ 8(d+17)]+ ... ded, Bed+TT = Sia[d+(n-1)(d+1)] sin (n(d+1)) Sin (d+11) = Sin [Qd + nd +n. T - d + TT] sin [nd+T] sin (2+11) = GIA [X (MAI) + TT (N-1)] Your [n(x+1)] Sin (d+ 17) = Sin [x+(n-1)(x+11)] sin (n(x+11)) Sos (2)