## UNIT- IV Hyperbolic function Definition: The hyperbolic functions are defined by i) Sin hx = ex ex => cosec hx = \frac{1}{\sinhx} = \frac{2}{\epsilon x - ex - e - x} ii) cos hx = extex =) sechx = wshx = extex iii) tan hx = $\frac{\sin hx}{\cos hx}$ = $\frac{e^{x}-e^{-x}}{2}$ | $\frac{e^{x}+e^{-x}}{2}$ = $\frac{e^{x}-e^{-x}}{2}$ = $\frac{e^{x}-e^{$ Lot how = $\frac{1}{\tanh x} = \frac{e^x + e^{-x}}{e^x}$ 802 4 3 4 4 8 th 1/2 x = 608 10 2 x Result: 1 Cosh2x - sinh2x =1 L.H. b 3 Cosh & sinh 2x = [exterx]2 [ex-e-x]2 $= \frac{(e^{x}+e^{-x})^2}{4} - \frac{(e^{x}-e^{-x})^2}{4}$ $= \frac{1}{4} \left[ (e^{x} + e^{-x})^{2} - (e^{x} - e^{-x})^{2} \right]$ = 1 [(ex+e-2x+ 2ex.e-x)-(e2x+e-2x-2exe-x) = 1 [eax + e-2x, 2-e-e-42] = = (4)

Soln

-R.H.S

Result: 2 Sin hax = 2 sinhx wshx asin ha cosh x =  $2\left[\frac{e^{x}-e^{-x}}{2}\right]\left(\frac{e^{x}+e^{-x}}{2}\right)$ R.H.J : = A [e 2x - e-2x] = = (e2x\_e-2x) Result: 3 ash'x +sinh2x = cosh 2x. (05h2x + SIn h2x = [ex + e-x]+[ex-e-x]2 L.H. 8:  $= \frac{(e^{\alpha} + e^{-\alpha})^2}{4} + \frac{(e^{\alpha} - e^{-\alpha})^2}{4}$  $=\frac{1}{4}\int (e^{x}+e^{-x})^{2}+(e^{x}-e^{-x})^{2}$ = \frac{1}{4}\left(\ell^{2\chi} + \ell^{-2\chi} + 2 \cdot \ell^{-\chi}\right) + FOR 421-34 MODE . (eax+e-2x-2exe-x) = 1 [(e<sup>2x</sup>+e<sup>-2x</sup>+2)+(e<sup>2x</sup>+e<sup>-2x</sup>2)] = 1 [e2x+e-2x+2+e2x+e-2x] = 1 [e2x+e-2x+e2x+e-2x] = 1 (2e2x+2e-2x)

Result:5

$$coshox = 1+ & sinh^2x$$
 $cosh^2x = \left(\frac{coshox+1}{2}\right)$ 
 $sinh^2x = \left(\frac{coshox-1}{2}\right)$ 

NOTE:

Timmediate consequence of definition.

 $1-sinhx = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$ 
 $2-coshox = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$ 
 $3-coshox = 1$ 
 $sinhox = 0$ 
 $4. N. k. T$ 
 $sinhox = 0$ 
 $4. N. k. T$ 
 $sinhox = 0$ 
 $cosox = 1 - \frac{0^2}{2!} + \frac{0^4}{4!} + \frac{0^6}{6!} + \cdots$ 

Relation between hyperbolic functions and

Relation between hyperbolic functions and circular trigonometric functions:

Theorem: (i) sin (ix) = isinhx

(ii) Cos (ix) = coshx

(iii) toun (ix) = i tan hx

Phoof of (i)

N.K.T. 
$$\sin \theta = \theta = \frac{\theta^3}{8!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!}$$

Put  $\theta = ix$ 
 $\sin(ix) = (ix) - \frac{(ix)^3}{8!} + \frac{i5x^5}{5!} - \frac{i'x^7}{7!} + \frac{ix}{2} + \frac{i^2x^3}{3!} + \frac{i5x^5}{5!} + \frac{ix^7}{7!} + \frac{ix}{2} + \frac{i^2x^3}{3!} + \frac{ix^5}{5!} + \frac{ix^7}{7!} + \frac{ix}{2} + \frac{ix^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \frac{ix}{2} + \frac{x^5}{4!} + \frac{x^5}{6!} + \frac{x^5}{6!} + \frac{x^5}{4!} + \frac{x^5}{6!} + \frac{x^5}{6!$ 

Price of (11) (iii) tan (ex) = sin(ix) Costax) = isinha ashx . tan (ix) = itan ha Example: 1 POR Corousponding to the formula coso + sin20 =1 cos20 +sin20=1 Put 0= Ix cos2(ix)+Sin2(ix)=1 [cos(ix)]+[sin(ix)]=1 [cos(ix)] + [sin(xa)] -1

[cos(ix)] + [sin(xa)] -1

[cos(ix)] + [sin(xa)] -1 Cos2hx - sin2hx =1 Example: 2 1 consider cos(A+B) = cosA cosB - sin A sin B Put A = ix B= i4 cos (ix +iy) = cos(ix) cos(iy) - sin(ix) sin(iy) cosi(x+y) = coshx. coshy - isinhx. isinhy cos h (x+y) = coshx. coshy + sin hx. sin hy

Inverse hyperbolic function: Definition: consider the funtion your har. This is a 1-1 onto map from R-SR : biven any yer there exists unique x such that sin hx = y we define X = sinh 'y. 11/4 Y= cos hx is a map from R->[1,00) . Both x and -x have the same image undex costox. Hence given by any y E [1,00) we can find a unique x such that los hx=y We define x=ash-'(y) and x is called the principal value of cosh-'(9). : The function y tan ha is a map from R-> (-1,1) given any YER of unique & suct that tan hx = y We define x = tan h-1(4). (i) sin h-1 (x) = loge (x+1/2+1) Theorems: (ii) cos h-1(x) = loge (x+1/x2-1) (iii)  $tan h^{-1}(x) = \frac{1}{2} log \left(\frac{1+x}{1-x}\right)$ over success of the son

Proof: (i) Let y=sinh-1(x) x = sinhy  $x = e^{y} - e^{-y}$ x both sides ey  $xe^{y} = e^{y} \left[ \frac{e^{y} - e^{-y}}{2} \right]$ 2xey = e<sup>2</sup>y - e<sup>-</sup>y y 2xey = e<sup>2y</sup> - 1 e24-2xe4-1=0 a=1, b=-2x, (=-)  $\chi = -67\sqrt{b^2-490}$ 29  $e^{4} = 2x \pm \sqrt{4x^{2} - 4CU} C - 1)$  $e^{y} = 2x \pm \sqrt{4x^2 + 4}$  $e^{y} = \frac{2x \pm 2\sqrt{x^2+1}}{2}$ ey = 21 ± 1/22+1 since e' is always the ey = x+ (x+1 Taking log on both sides, y = loge (x+ 12+1)

Proof: Let y= cos h (x) x = cos hy x= e4+e-4 x both sides ey  $xe^{y} = e^{y} \left[ \frac{e^{y} + e^{-y}}{2} \right]$ 2xey = e2y+1 e29- Qxe +1=0 a=1, b=-2x, (=1  $X = \frac{b + \sqrt{b^2 - 4ac}}{2a}$  $e^{y} = \frac{2 \times t \sqrt{4 x^2 - 4}}{2}$  $e^{y} = \frac{2x + 2\sqrt{x^2-1}}{2}$ e y = x ± [x2-1 since ey is always +  $\sqrt{2}$   $e^{y} = x + \sqrt{x^2 - 1}$ Taking log on both sides, y = loge (x+ (x2-1).

Proof:

(iii) Let 
$$Y = tan h^{-1}(x)$$
 $x = tan h y$ 
 $x = e^{y} - e^{-y}$ 
 $x(e^{y} + e^{-y}) = e^{y} - e^{-y}$ 
 $x(e^{y} + e^{y}) = e^{y} - e^{-y}$ 
 $x(e^{y} + e^{y}) = e^{y} - e^{-y}$ 
 $e^{y} - e^{y} - e^{-y} - e^{-y}$ 
 $e^{y} - e^{y} - e^{y} - e^{y} - e^{y}$ 
 $e^{y} - e^{y} - e^{y} - e^{y}$ 
 $e$ 

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P. T (wshx & sin hx) = coshnoca sinh noc
    Solas
    2.4.5: (cosh ax & sinhox) = [cos (ix) - i sincex)
                 = [cosn(be) - i kinn(1x)]
    by De-Moiver's Hours
    = [cosi(noc) vi sin i(noc)]
           (ws hx + sin hx) = coshnx + sin hnx
                    Hence proved 1.19-5 = R.19-5
2. P.T: 1+tanhx = cosh 2x + sinh2x
        1-tanhx
    oln:

1-tanhx = \frac{1+\frac{\sin hx}{\cos hx}}{1-\tanh x} = \frac{1+\frac{\sinh x}{\cos hx}}{\cos hx}
   Soln:
             = \frac{\cosh x + \sinh x}{\cosh x} \times \frac{\cosh x}{\cosh x} =
            coshx + sin hx
                         Coshx - sinhx
                      = cosha + sin ha x cosha + sin ha
                                         coshoc thin ha
                   basha sinha
                  = (coshac + sinhae)2
 cosh 2 + Sinh 2x = cosh 2x forh 2x + Sinh 2x + 2 sinh 2x cosh 2x [cosh 2x + Sinh 2x + 2 sinh 2x cosh 2x]
                          cosh2x - sinh220
            1+ tanha = los han + sin hax
                    Hence proved
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3. If cos (xiey) = cos O 1 Esino PI
      cos 201 + cosh 24 = 2
     Soln:
        los (xxiy) = coso+ i mo
     los(x) los(iy) - sin(x) sin (2y) = 1050 + 2 xin 0
     cos x coshy - isinx win hy = wis 0+ i rino.
       Equating real & imaginary terms
       Cosx coshy = coso -> 0
     - sina winhy = sind > 0
    (D=) cos20 = cos2x cosh2y
                               ed not -1
    @=> Sinto = 8in2 x 8inh24
              0+0
    Cos2xcosh2y + 85n2x 85n h2y = cos20 + 18in20
cos2x cosh2y + (1-cos2x) sinh2y = 1
   cos2x cosh2y + sinh2y - cos2x 181 h2y = 1
   Cos2x (cosh2y-sinh2y) + sinh2y = 1
  1052x + sin2y = 1
  1+cos2x + 1 (cosh2y-1)=1
  1+cos2x + cosh2y-1=2
  COS 20 + GOSh 24 = 2
          Hence proved.
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4. If sin(Orig) = tandrises & P.T COS 20 COS h2y = 3 Soln! sin (0+iy) = tand+i seld sin o cos (i4) + cos o sin (i4) = tanda i seed sino coshy +icoso sinhy = tand+i secd Equating real 4 imaginary lenns. sino cos hy = tan d -> 0 coso sin hip = sec d -> @ Using this term 1+tan2d = 15ec2d 1+ sin 20 cosh2 4 = cos20 sinh24 1+ (1-60520) COSh2 41= COS20 Kinh24  $1 + \cosh^2 \psi - \cos^2 \alpha \cosh^2 \psi = \cos^2 \alpha \sinh h^2 \psi$  $1 + \cos^2 \psi - \cos^2 \theta + \cosh^2 \psi = \cos^2 \theta + \sinh^2 \psi = 0$ 1+ cosh24 - cos20 (cosh24 + sinh24) = 0 1+ cosh24 - cos20 (cosh24)=0 1+ (1+ coshay) - (6) 1+ cos20) cosh2 4 = 0  $8 + 1 + \cosh 2 \psi - \cosh 2 \psi - \cosh 2 \psi = 0$ 3 - Ws 2 0 Wsh2 4 = 0 Cos 20 Cosh 2 4 = 3 Hence phoved.

5. If tan (atib) = x + 2y P.T x = sinza Soln: xtiy = tan (atib) 21/14 = sin (a+ib) los (arib) - sin (a+ib) x cos (a-ib) cos (a+ib) cos(a-ib) xiy = cos(a+ib) cos(a-ib) For understanding purpose. sin (A+B) = kinAcosB + cosA &inB Sin (A-B) = MinAcosB & COSA MINB By adding these two we get, Sin(A+B)+Min(A-B) = 2MinA COSB COS(A+B) = COSACOSB - MINAMINB WS (A-B) = aSA. WSB. + SINA KINB By adding these two we get, 05(A+B)+(55(A-B) = 2 COSA COSB : X+ iy = &in (a+ib+a-ib) + sin (a+ib) - a+ib) cos (arib+a-ib) + this (arib-arib) sin & a + kin (2ib) los 2a + los (dib) = sin aa + isin hab actin Cossa + cos hab

= tanh B x los2hB - SinhB wshB land losa Now X and = 2, 2 Sind wsd = Sln2hBi Sin 2d tan(A+B) = loin 2hB cosec 2d Hence proved. 8. Sin hax = atanhx
1-tan2hx soln: N.K.T sin20 = 2tano put o = ix sin 2 (ix) = altan(ix) sin i 2hx = 2 i tanhx Sin 2hx = atanh 2 1-tan2hx Hence proved.

9. cos hax = 1+tan2hx soln: N. K.T COS 20 = 1-tan20 put o = ex (05 2 (ix) = 1 - 1 tan2 (ix) 1+0 tan2(ix)  $\cos 2h \propto = \frac{1 - i^2 \tan^2 h x}{1 + i^2 \tan^2 h x}$  $\cos 2hx = \frac{1 + \tan^2 hx}{1 - \tan^2 hx}$ 10. tan (x+iy) separate into real & imaginary part: Soln:  $tan(x+iy) = \frac{sin(x+iy)}{cos(x+iy)}$ \_sin(x+iy) x los(x-iy) cos(ntiy) cos(x-iy) = Sin(x+iy) cos(x-iy) cos(x+iy) ws (x-iy) = sin (x+iy+x-iy)+sin (x+iy-x+iy)
cos(x+iy+x-iy)+ cos (x+iy-x+iy) = sin(2x) + sin (aiy) 65(2x)+65 (284) = Sin 2x + i sin 2 hy cos 2x + cos ahy Equating the real & imaginary part sin 2x los 2 hy i y = sin 2 hy x-Sin2x

11. P.T tanh 32 = 3tanh x + tan3hx 1+3 tan2 hac Solo:  $W \cdot K \cdot T : \tan 30 = \frac{3 \tan 0 - \tan^3 0}{1 - 3 \tan^2 0}$ Put 0 = ex  $tan 3(ix) = \frac{3 tan(ix) - tan^3(ix)}{1 - 3 tan^2(ix)}$ itan3hx = 3itanhx - itan3hx 1-3i2 tan2 hx  $i tan h 3x = 3.i tan h x + i tan^3 h x$   $1 + 3 tan^2 h x$ 1+ 8 tan2 hx i (tan h3x) = i(stanhx) tan3hx)  $tan h32 = 3tan hx + tan^3 hx$ 12. To find real & imaginary points of cosh (xiiy) cosh(x+iy) = cos i (x+iy) = cos(ix + i2y) = cos (ix-y)  $= \cos(ix)\cos y + \sin(ix)\sin y$ = cos(xx) cos y + i sin hx sin y Real part = los ha cos go Imaginary part = sinhx sin y.

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13 To find rual & imaginary posts of cost (14i)
   Solo:
   (MSh(H2) = (M) 2 (142)
            =(03(1+12)
            = (05(2-1)
            = losi cos 1 A Libiscon 1
            = tosh tos + sinh sin
  Real pant : cosh cos
  Imaginary part: sin h kin
To find heal and imaginary of sec (2+44)
 Sec(x+14) = (05(x+iy)
         - cos(x+iy) x cos(x-iy)
 -\frac{2}{a}\cos(x-iy)\cos(x-iy)
          = 2 [cos x cos (iy) + sinx sin(iy)]
            cos(x+iy+x-iy)+ cos(x+iy-x+iq)
          = 2 [cosx coshy + isinx sin hy]
           (05(2x) + cos(2)iy)
          = 20052 toshy + isinxsinhy
                            050x)+(05)
          (05(22)+605(hy)
Real part x= 2005x cos (hy)
Imaginar part y sinx sinhy
               cos (ox) + cos (thy)
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15. To find real & imaginary part of cot (xxiy)
                           Soln.
                          (ot (x+iy) = (05(x+iy)
                                                                                           = \frac{\cos(x+iy)}{\sin(x+iy)} \times \frac{\sin(x-iy)}{\sin(x-iy)}
                                                                                           \frac{-2}{2}\cos(x+iy)\sin(x-iy)
\sin(x+iy)\sin(x-iy)
                                                                                        = Sin(x+iy+x-iy)+sin (x+iy-x+iy)
                                                                                                       cos (x+ivs sixtex+ivs) successfully sixtex-iy)
                                                                                                Sin (2x) + Sin (2iy)

(05(2iy) - cos2x

Sin (xxiy) & sin(xxiy)
                                                                                          - Sin(2x) + isin hay
Sin(2x) + isin hay
Sin(2x) + isin hay
                                                                                           - Sin(2x)

                   Real part = sin(2x)
                  Imaginary part = win hay - cos2x
16. Find the value of sinh (3/4)
                   Sin h-1 (3/4)
                 M.K.T: Sink'x=log (x+v2+1)

put x=3/4
                                             Smh (3/4) = log (3/4 + \(3/4)^2+1
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= 69 (3+1941 = log ( = + 176) - 69 (3 + 5) = log ( \$ ) Sin h 1 (34) - 609 2 17 P.T. WSh'(2) = log [x+1/22-a2] soln: WKT: 605h-12 = 609 [20+ 122-1] Put x = 2 65 h (8)= 69 [2+ 120-1] = log [ 2 + (22-a2 ] = log [x+Vx2-a2 18. PT: Sinhox = asinhx+asin & x soln: N.K.T: Sin 80 = 8sino - Asin30 put o : 2x sin acix) = asincix) - 4 sin3 (ive) isin hax = 3ikin hx - 4i3 sin3 hx isinhax = 3isinhx + Ais sin3hx i (sinhax)=i(asinhx + 4 sin3hx) sin hax = 3 sin hx + 4 min hx

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19. Separate real & imaginary part of tan (d+ ip)
         24 iy = tan ( ( x + ip)
       tan(x+ky) = x+kp.
     2x = (x+iy) + (x-ig)
   2iy = (x+iy) - (x-iy)
      tan (2x) = tan [(2+iy) + (x-iy)]
      tan 2x = tan (x+iy) + tan (x-iy)
    tan (A+B) = tan A+tan B
1-tanAtanB
     \tan(2x) = \frac{\tan(x+iy) + \tan(x-iy)}{1 - \tan(x+iy)\tan(x-iy)}
     tan 8x = \frac{(\alpha+i\beta)+(\alpha-i\beta)}{1-[(\alpha+i\beta)(\alpha-i\beta)]}
      =\frac{\partial^2}{1-(\lambda^2-(i\beta)^2)}
tan \partial x = \frac{\partial^2}{1-(\lambda^2+\beta^2)}
    8x; tan (1-(2+82))
        (x= \frac{1}{9} \tan' \left( \frac{1-(d^2+\beta^2)}{1}
                           Ania Kry Valle
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diy = (x+/y) - (x-iy) tan (oxy) = tan [(xxxy) - (x-xy)] tan(A-B) = tan A - tan B tan(Diy) - tan(x+iy) - tan(x-iy) (dtip)-(d-ip) 1+[(d+ip)(d-ip)]  $\tan 2iy = \frac{2i\beta}{1+(2^2+\beta^2)}$ Diy = tan - ( 21B (1+ (2°+B2)) y= 1/2 tan-1 (1+ (d2+B2)) 20. Separate into real & imaginary parts: i) Sin-1 (cos o + i sin o) let x+iy = sin (coso+isino) sin(x(tiy) = coso + isino. Sina cos(iy) + cosasin (iy) = coso + i sin o sinx coshy + i cos x sin hy = cos o + i kina Equating the real & imaginary terms cos 0 = vinx cos by -> 0 sin 0 = cosx sinky

squaring & adding 040 605°0+ 400°0 = 100°× 605° hy + 605°× 100° hy 1 = Sin2x (11/6/n2/hg)+ (1-16/n2x)/6/n2/hg 1 = 18in2x 4/8i n2x xin2 by + 18in2 by -Min'x Min' hy 1 = 85n2x + 85n2 hy 1-8in2x = 8in2 hy as x = sinhy -> B 8 in @ 65n 0 = 605 x . 605 2 18in 0 = Cos2 x 8in 0 = (cos x)2 cos x = Vino x = 605-1 VISINO sin hy = cosx = Vino sin hy = 1/600 y= sinh : [Vino] Real pour x = los -1 (Vásino) Imaginary part y= Sinh-1(18ino).

ii) Sinh (daip) soln: 2+iy = Sinh (2+ip) - Sin (i (d+iB)) = + sin (id-B) =-i [sinkly) cos. p - 603(id) sin p] =-i [ikin ha losp - losh a king] 2+iy = sin ha cosp+ i ws hasing. Real part x = 18inh & cos B 2maginary part 4 = cosh & sin B. iii) tank (+i) Soln: Let x+iy = tan h(1+i) = Sin h (1+i) Shha cosh (1+i)  $=\frac{1}{i}\frac{\sin(i(Hi))}{\cos(i(Hi))}$   $=-1\int_{-1}^{1}\int_{-1}^{1}\sin(i-1)T$ =- 2 | Sin (i-1)  $=-i\left[\frac{\sin(i-1)}{\cos(i-1)}\times\frac{\cos(i+1)}{\cos(i+1)}\right]$  $=-i\left[\frac{2\sin(i-1)\cos(i+1)}{2\cos(i-1)\cos(i+1)}\right]$  $=-i\left[\frac{\sin(i-1+i+1)+\cos(i-1-i-1)}{\cos(i-1+i+1)+\cos(i-1-i-1)}\right]$ 

$$=-i\left[\frac{\sin 2i + \sin (-2)}{\cos 2i + \sin (-2)}\right]$$

$$=-i\left[\frac{i \sin 2i + \sin (-2)}{\cos 2i + \sin (-2)}\right]$$

$$=-i\left[\frac{i \sin 2i + \sin (-2)}{\cos 2i + \sin (-2)}\right]$$

$$2+iy = \frac{\sin 2i + \sin 2}{\cos 2i + \sin 2}$$

$$2+iy = \frac{\sinh 2}{\cos 2i + \cos 2}$$

$$Real part  $x = \frac{\sinh 2}{\cosh 2 + \cos 2}$ 

$$2\sin 2i + \sin 2i + \cos 2i$$

$$\cos 2i + \cos 2i$$

$$\cos 2i + \cos 2i + \cos 2i$$$$

If 605d + 185nd= 605(0+16) PT sin20 = + sind C say had Soln: (05d+i&nd= cos(0+iq) cosd+ikind = coso costiq) - sino sin(iq) losd + i soin d = los o cosh & - i sino sinh d Equating real & imaginary terms. cosd = coso cosh of cosd = cosho ->0 0000 sind = 18snossinho sind = sin ho -> 0 Now squaring & subtract  $\frac{\cos^2\alpha}{\cos^2\theta} - \frac{\sin^2\alpha}{\sin^2\theta} - \cosh^2\theta - \sinh^2\theta$ cosa sin2 - sin2 wso coso sino (1-sin2)sin20 - sin2 (1-sin20)= 600 (1-sin20) sin20 sin20-8in2d sin20 - 8in2 + 8in2d 8in20 = sin20 - sin20 (sin20 148 AMM 3 46 x 23 m2

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sing - Mind king o - King + King & King o = King o - King o
   Sin 20 - 18in'd = 18in 20 - 18in 40
   sin'0-sin'd -sin'0+sin'0=0
         Sin 40 - 18in 2d = 0
         Siny a = 181n2d
   Taking square mots on both tides
          SINO = ± Mind.
2. If (x+iy) = tan (A+1B). P.T x2+y2+ 22 wt 21 =1
                    ing a forestropy of the page
   Soln:
      xxiy = tan (AtiB)
      x-iy = tan (A-iB)
   \cot 2A = \frac{1}{\tan 2A}
\tan (A+B) = \frac{\tan A + \tan B}{1 - \tan A + \tan B}
   Cot &A = 1-tanAtanB
            = 1-tan(A+iB) tan (A-iB)
               tan (AtiB) + tan(A-iB)
            -1-(x+iy)(x-xy)
x+iy+x-iy
            =!-(x2-xiy+xiy+y2)
     \cot 2A = 1 - \frac{\alpha^2 - y^2}{2\alpha}
    2x Lot 2A = 1-2e2-42

x2+42 + 2x Cot 2A = 1
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Roju) tan (xxxiy). Let x+iy = tan-1(x+iy) Soln: tan(Rtiy) = Xtiy A= X+iY B=x-dy 2x = (x+iy) + (x-iy)2iy = (x+iy) - (x-iy) tan(2x) = tan((x+iy)+(x-iy)). tan(2x) = tan(x+iy) + tan(x-iy) tan (A+B) = tanA+tanB 1-tanAtanB tan(2x) = tan(x+iy) + tan(x-iy)1- (tan(x+iy) tan(x-iy))  $tan 2x = \frac{x+iy+x+iy}{1-[(x+iy)(x-iy)]}$  $tan 2x = \frac{2x}{1-(x^2+y^2)}$  $&x = tant \left(\frac{&x}{1-fx^2+4^2}\right)$ 2 = \frac{1}{2} \tan \left(\frac{2x}{1-(x^2+y^2)}\right) 2iy = [x+iy) -(x-iy) tan(2iy) = tan[(x+iy) - (x-iy)] tan (2iy) = tan(x+iy) - tan(x-iy)

tan (A-B)= tanA-tanB

1+ tanA tanB tan (Riy) = tan (x+iy) - tan (x-xy) 1+ Stan(x+ky) + tan(x-ky) - xiy-xiy 1+ [(x+ iy)(x- iy)]  $tan 2iy = \frac{2iy}{14(2^2+4^2)}$ 2iy = tan- 1 (xetip) y= = tan-1 (24 1+(x2+42)) p.T u=log tan [4+2] y cos hu=secro Sola: Cos hu = 860 0 u= losh-1 (1860) D.K.T: cosh-(x) = log(x+1221) :. u = log (seco + v sec20-1 u= log [ toso + Itan = 0] u=log[coso +tano] u= 69 [coso + sino] Then put soin 20 = 2tan 0 85n 0 = 2 tan 0/2

Put 
$$t : tan 0/2$$
 $sho = \frac{2t}{1+t^2}$ 
 $(oso = \frac{1-t^2}{1+t^2})$ 
 $u : log \left(\frac{1-t^2}{1+t^2} + \frac{2t/1+t^2}{1+t^2}\right)$ 
 $= log \left(\frac{1+t^2+2t}{1-t^2}\right)$ 
 $= log \left(\frac{(t+t)^2}{1-t}\right)$ 
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 $u : log \left(\frac{tan \sqrt{u} + tan 0/u}{1-tan \sqrt{u} + tan 0/u}\right)$ 
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 $u : log \left(\frac{tan \sqrt{u} + tan 0$ 

```
tand = - tan x tan hy
                       tanhy? - tan &

tan x

y = tan h [-tan x]
white \int x = \frac{1}{2} \log \left[ \frac{1+x}{1-x} \right]
    : Y= \frac{1}{2} log [\frac{1-\tan\d/\tan\d}{1+\tan\d/\tan\d}]
                        Y= \frac{1}{2} \log \big \tan\chi - \tan\chi \ta
                               Y= 1/2 log [tanx-tand]
                                 = \frac{1}{2} \log \frac{\sin x}{\cos x} - \frac{\sin x}{\cos x}
                                                         = \frac{1}{2} \log \left[ \frac{\sin x \cos x - \sin x \cos x}{\cos x \cos x \cos x} \right]
\frac{\cos x \cos x \cos x}{\cos x \cos x}
                                  = 1 log [sinx ws & - &fndws x]

sinx cosd + Mndcosx
                                      = / log [ sinxws
                                                            = 1 log (sin (2(-2))
```

5 P.T 
$$\frac{1}{8}$$
 [cin 0 + Min ho] =  $\frac{0}{12} + \frac{0}{12} + \frac{0}{12} + \frac{0}{12} + \frac{1}{12} + \frac{1}{12$ 

```
I. If xij = cos(u+iv) where x, y, u, v are real
    P.T: D)
 1) (1+x)2+42=(coshv+cosu)2
11) (1-22+42 = (coshv - cosus)2.
    Soln
        Let x+iy = cos(u+iv)
            xxiy = (asu costiv) - sinusin(iv)
                   = cosucoshv - i linukinhv.
      Equating real & imaginary parts
           2 = cosucoshur => 22 = cosºucoshev
           y=-sinusinhv=>y2=sin2usinh2v
  (1+x) 2+ y2 = (1+cosucoshv)2+ (sin2u sin h2)
             = 1+ cos²ucosh 2 + a cosu cosh v + sin²u sinh3
             = 1+ cos²u cosh² + 2cusu cushv + (1- cos²u)
             = 1 + \cos^2 u \cosh^2 u + 2 \cos u \cos hv + \cosh^2 v - 1
                                - cos24 wsh2V+ cos24
             = wsh2v + ws2 u + 2 wsu wshv
(1+x)2+y2 = (loshv+wsu)2
1 (1-29+42=(1-60sucoshv)2+(sin2u sinh2v)
          = 1 + cos²u cosh²v + Quosu coshv + sin²u sinh²v
          = 1+ 605° u cosh² v - 2 iosucosh v + (1- 605° u)
                                        (cosh2v-1)
          =1+60524 605h2v - 26054605hv + 605h2v -1
                                   - Losu wish 2 + 4052 u
          = 605 h2 V + cos2 u - 2 cos u coshv
```

(1-4)2+y== (005 Av- 6054)2 8. (ot h-1(x) = 1 log [201] Soln : Put y= cot h-1(x) a = cot h (y)

a = cot h (y)

sin hy  $\chi = \frac{e^{4} e^{4}}{e^{4} - e^{-4}}$  $x = \frac{e^{y} + e^{-y}}{e^{y} - e^{-y}}$ x(ey-e-y) = ey +e-y  $xey-xe^{-y}=e^{y}+e^{-y}$ xey-ey = e-y+xe-y ey(x-1) = e-y(x+1) ey (2(-1) = 2(+1) ey.ey = 2+1 e 24 = 27+1
2-1 Taking log on both sides  $ay = log \left[\frac{x+1}{x-1}\right]$ y= 1 log [2(+1] ath (x) = = 1 log [ 2+1]

9. P.T. With 
$$\left[\frac{x^{2}+1}{x^{2}-1}\right] = \log 2$$

W.K.7: Lot  $h^{-1}(x) = \frac{1}{2} \log \left[\frac{x+1}{x-1}\right]$ 

Lot  $h^{-1}\left[\frac{x^{2}+1}{x^{2}-1}\right] = \frac{1}{2} \log \left[\frac{x^{2}+1}{x^{2}-1}\right] + \frac{1}{2} \log \left[\frac{x^{2}+1}{x^{2}-1}\right] + \frac{1}{2} \log \left[\frac{x^{2}+1}{x^{2}-1}\right] + \frac{1}{2} \log \left[\frac{x^{2}+1+x^{2}-1}{x^{2}-1}\right]$ 

$$= \frac{1}{2} \log \left[\frac{2x^{2}}{2}\right]$$

$$= \frac{1}{2} \log x^{2}$$

$$= \frac{1}{2} \log x$$

Lot  $h^{-1}\left[\frac{x^{2}+1}{x^{2}-1}\right] = \log x$ 

Lot  $h^{-1}\left[\frac{x^{2}+1}{x^{2}-1}\right] = \log x$ 

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