The Algebra of Limits. TER In This dection ne prova a few simple theorems to sequences which are very useful in Calulating limits of sequences. Theorem 8.8 17 (an) -3 a and (pn) -3 p then (an+bn) -1 atb. broot: Let E-10 be given. Noco lanton-a-bl = lan-a +bn-bl & = lan-al + | bn-b) -10) since (an) sa, there exists a natural number n. such that 1an-a1 = = e for all n = n, . -> (2) since (bn) -> b, there exists a notural number no 1pv-pJ 5 = 5 6 pr opp 500 - 7 (2) Top w: wax {v. vog}. Then 1 an +bn -a-b/2 = + = = E goe of use (by 1,2 & 3). : (an+bn) -1 a+b. Note: .. similary we can Prove that (an-bn) -)

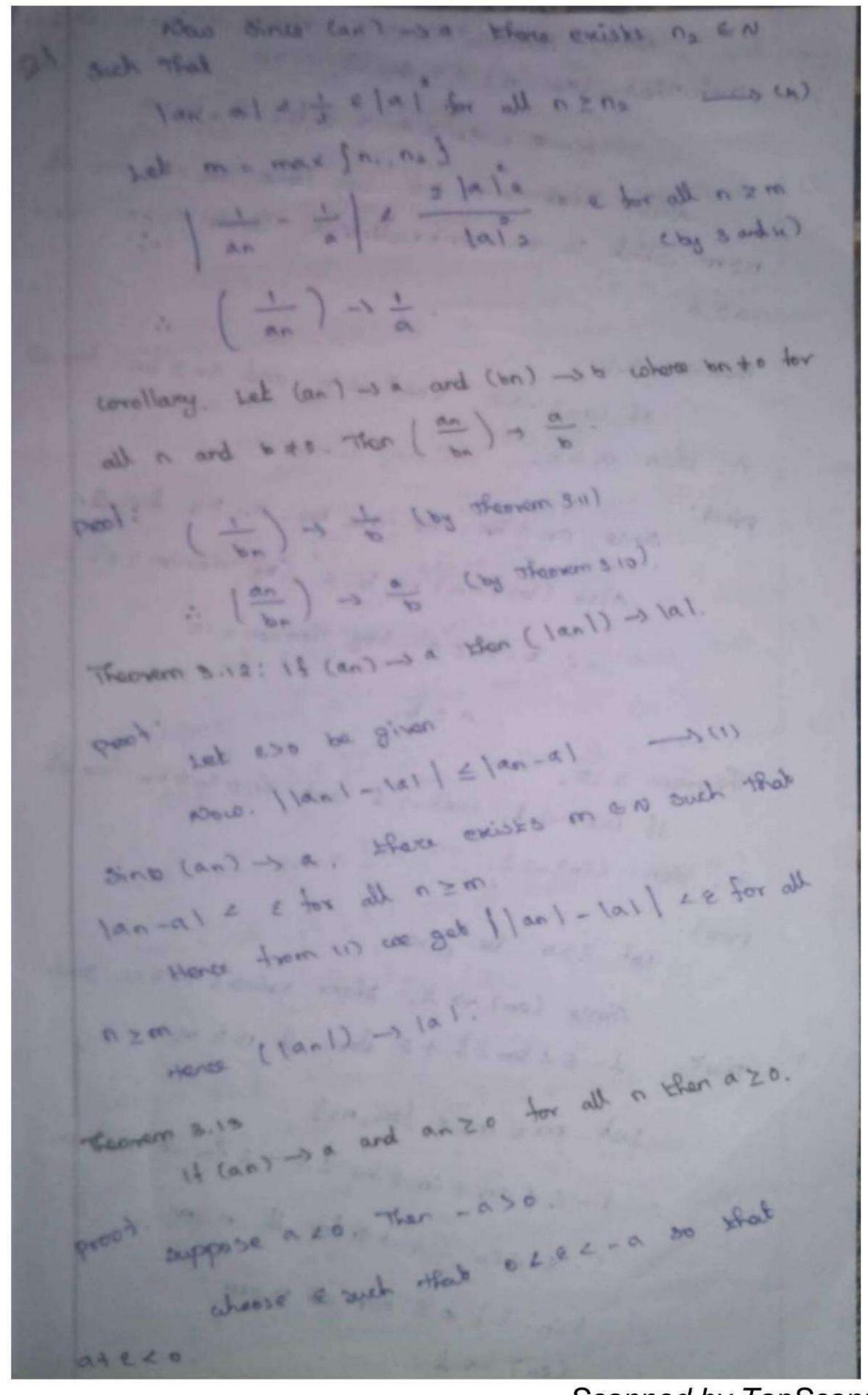
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if (an) -) a and KER Hon (Kan) -> Ka. Theorem 8.9 11 K=0. (Kan) is the constants sequence 0.0.0. and here the result is trivial. Now. Jel x 40. Let E>0 pe giver. since (an) -1 a, there exists men and that 1an-a12 @ IXI : I kan-kal LE for all n zon by ( and a) · ( Kan ) -> Ka if (an) -> a and (bn) -> b then (an bn) -> ab Theorem 3.10. proof let e > 0 be given Now. lander-abl = lander, and rand-abl F. 1 aupu - aupl 1 /aup-ap) = lanl/bn-bl + 16/1an-al -sur Also since (an) -300, (an) is a bounded sequence. by shower 3.2) ... There exists a real number kso such that lant & k for all n. \_\_\_\_\_ > [2]. using and can use got lanbn-abl = x /bn-b/ + 16/1an-al -> (3)

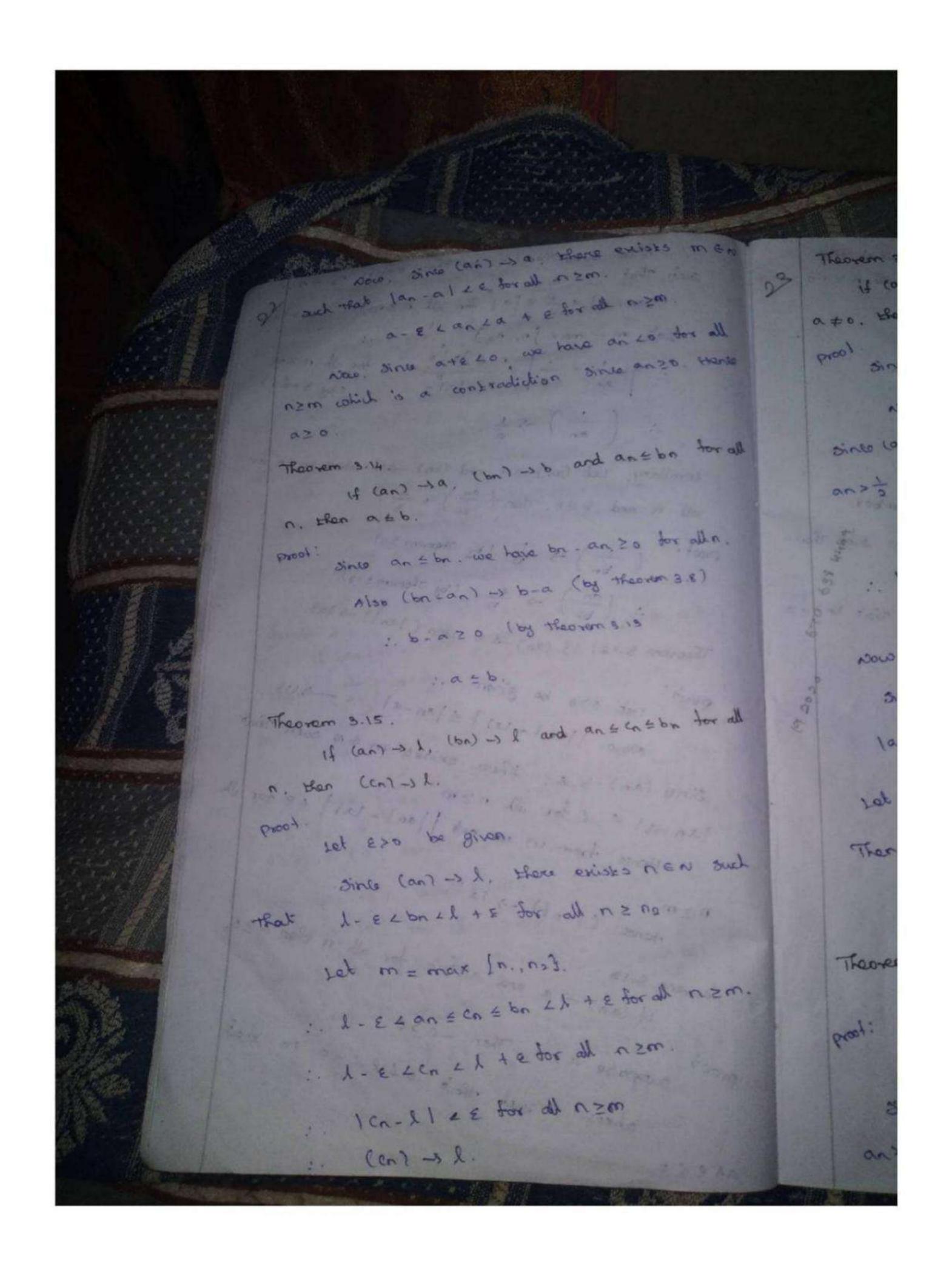
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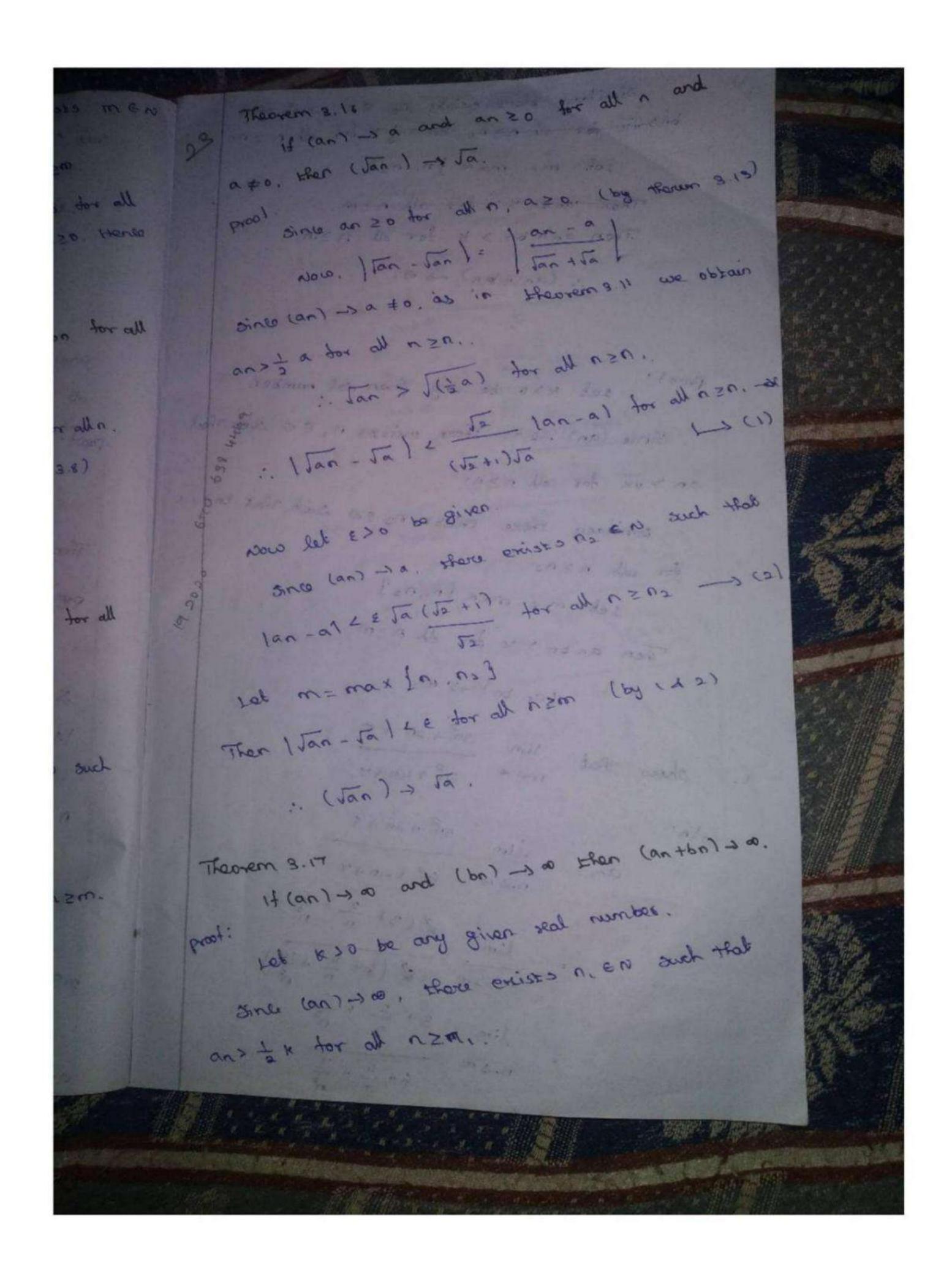
relieved number n. such that 194 - 41 - 2101 Jac of usau. - 18 Since (ba) - 1 b, Hora Exists a radiual runby sel moment for no 3. Then Janon-ablek (= ) +101 (= 2/61) = 8 for all n 2 m ( by 3. 4 and 5). Thomas III If (an) -) a and an +0 for all n and a +0. Theorem 2.11 Bon ( an) -> tel Eso be given we have tan all tana les tantials noois, a to Hence (a) >0. Sino (an) -) a work exists n. En such that Jan-al 2 - 1 al for all is 200.00 Henre Van 7 > 1 /al for all n 2 n. using (1) and (2) we gob 

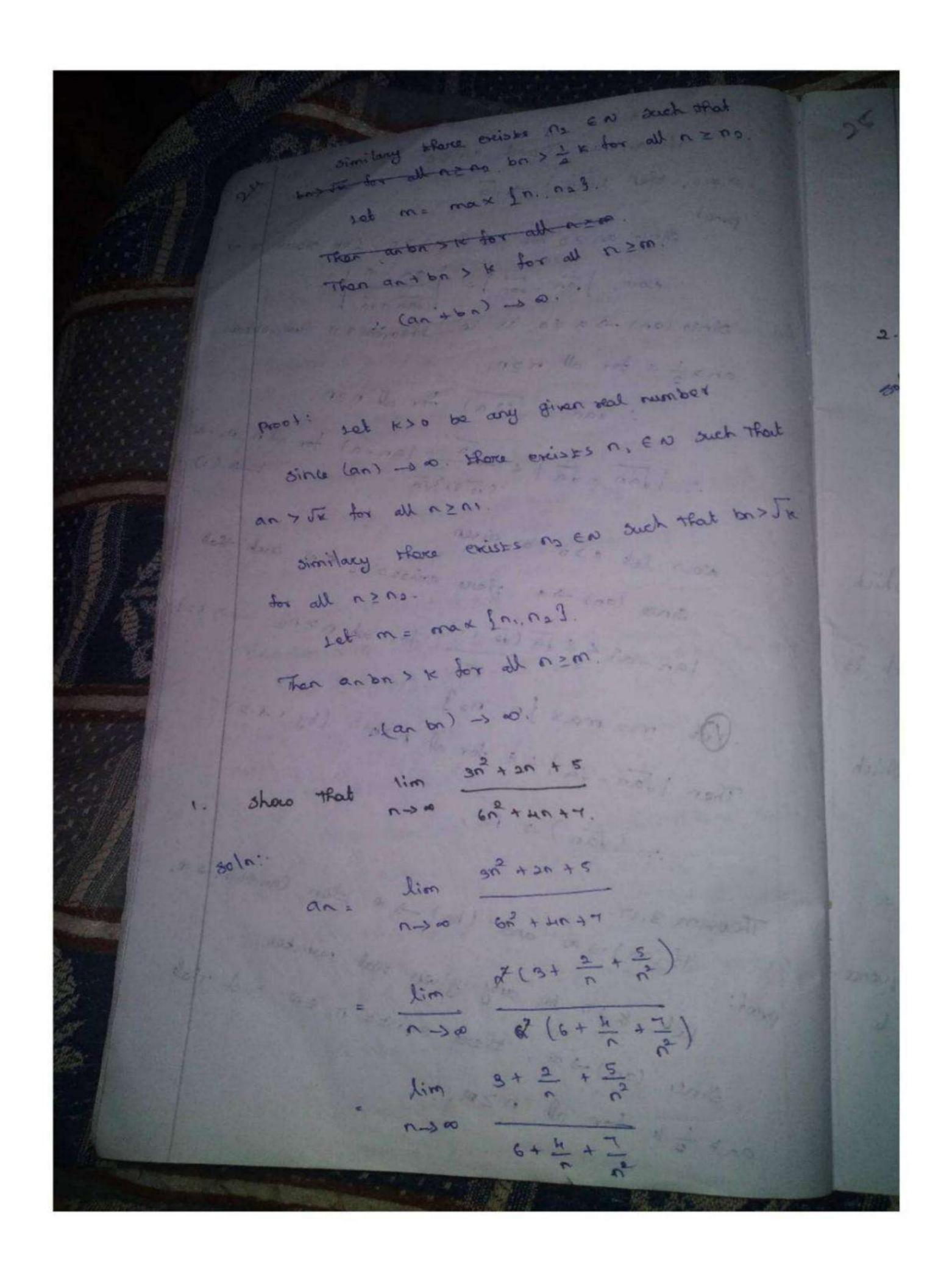
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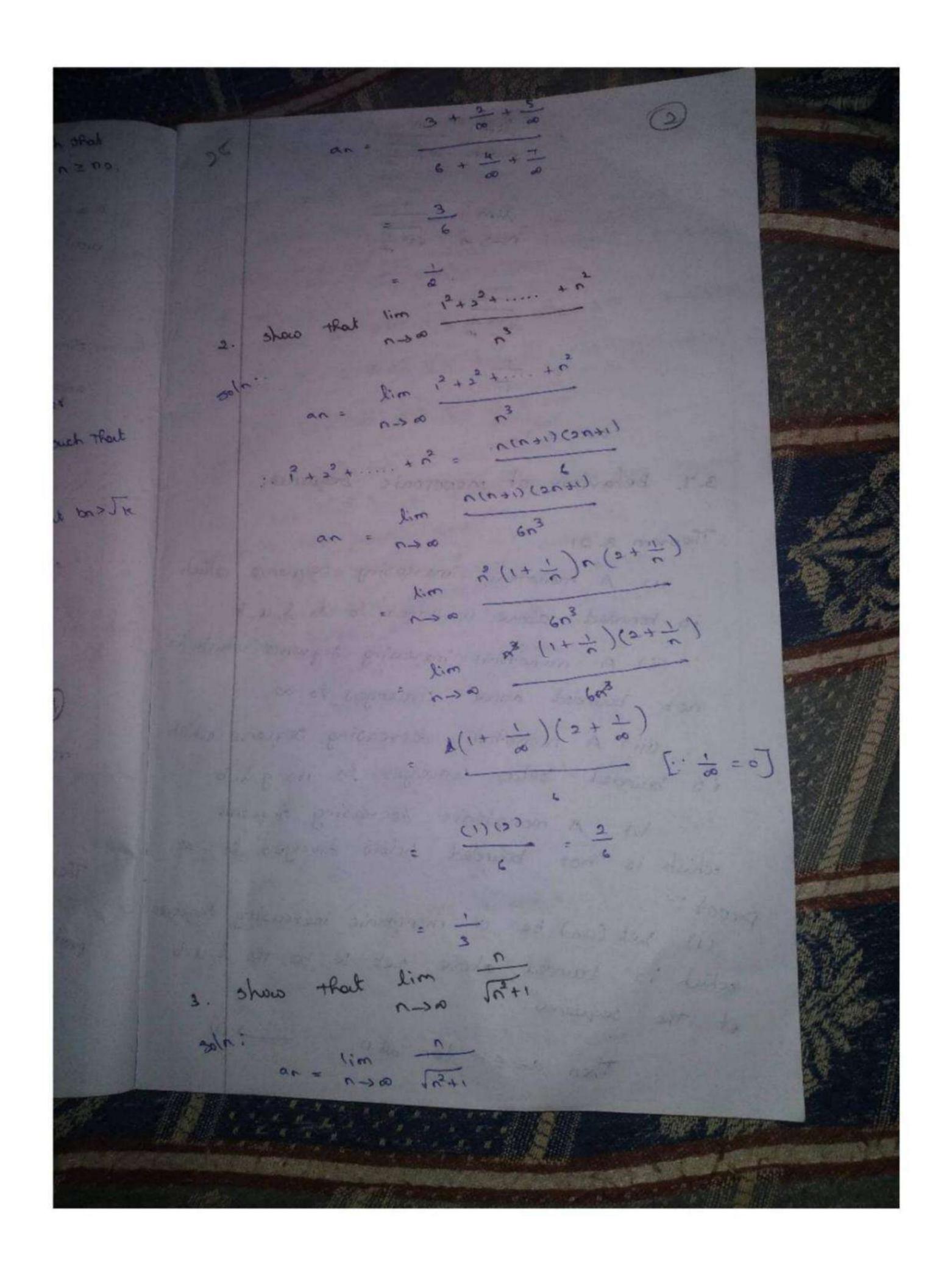
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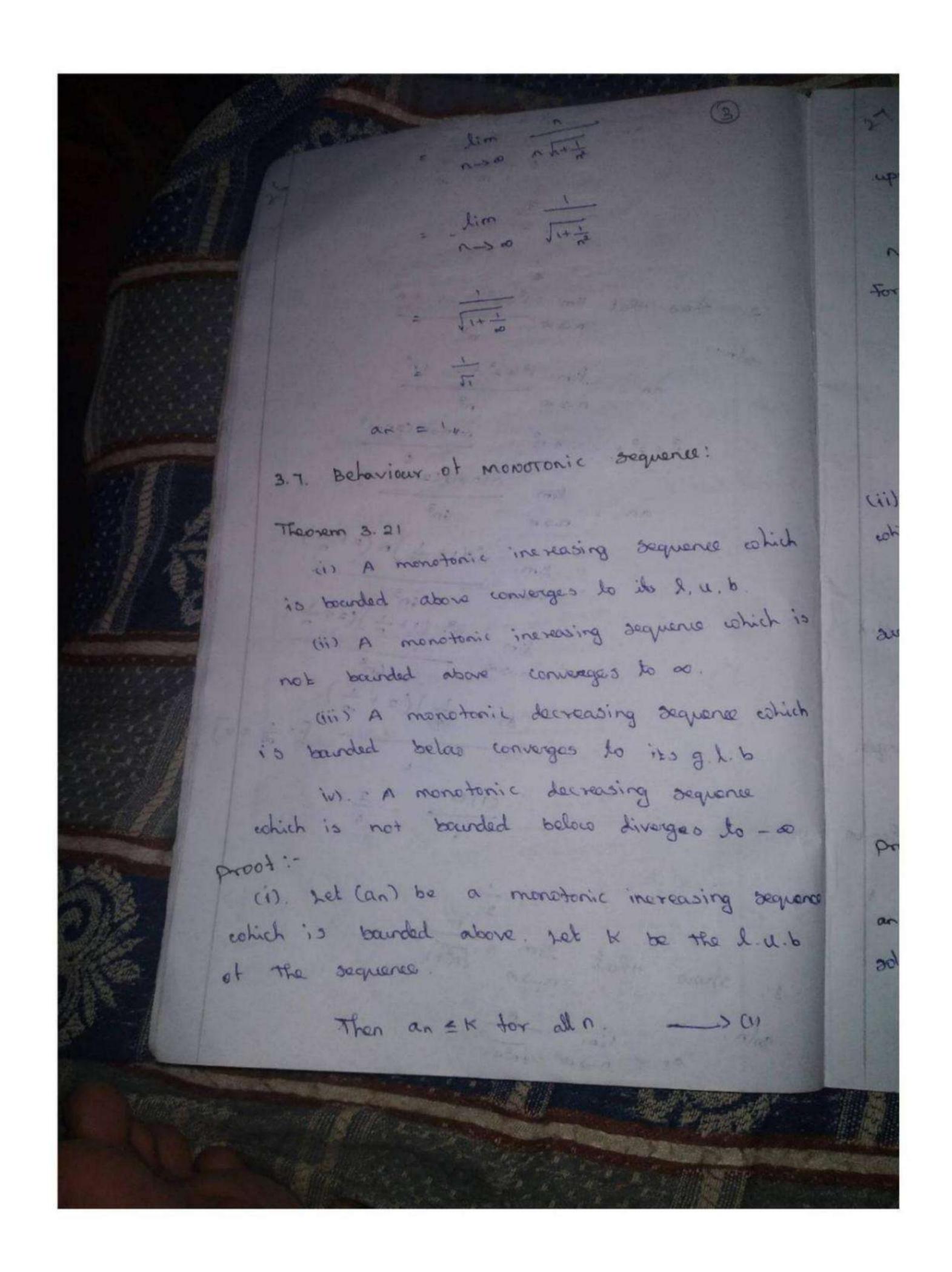


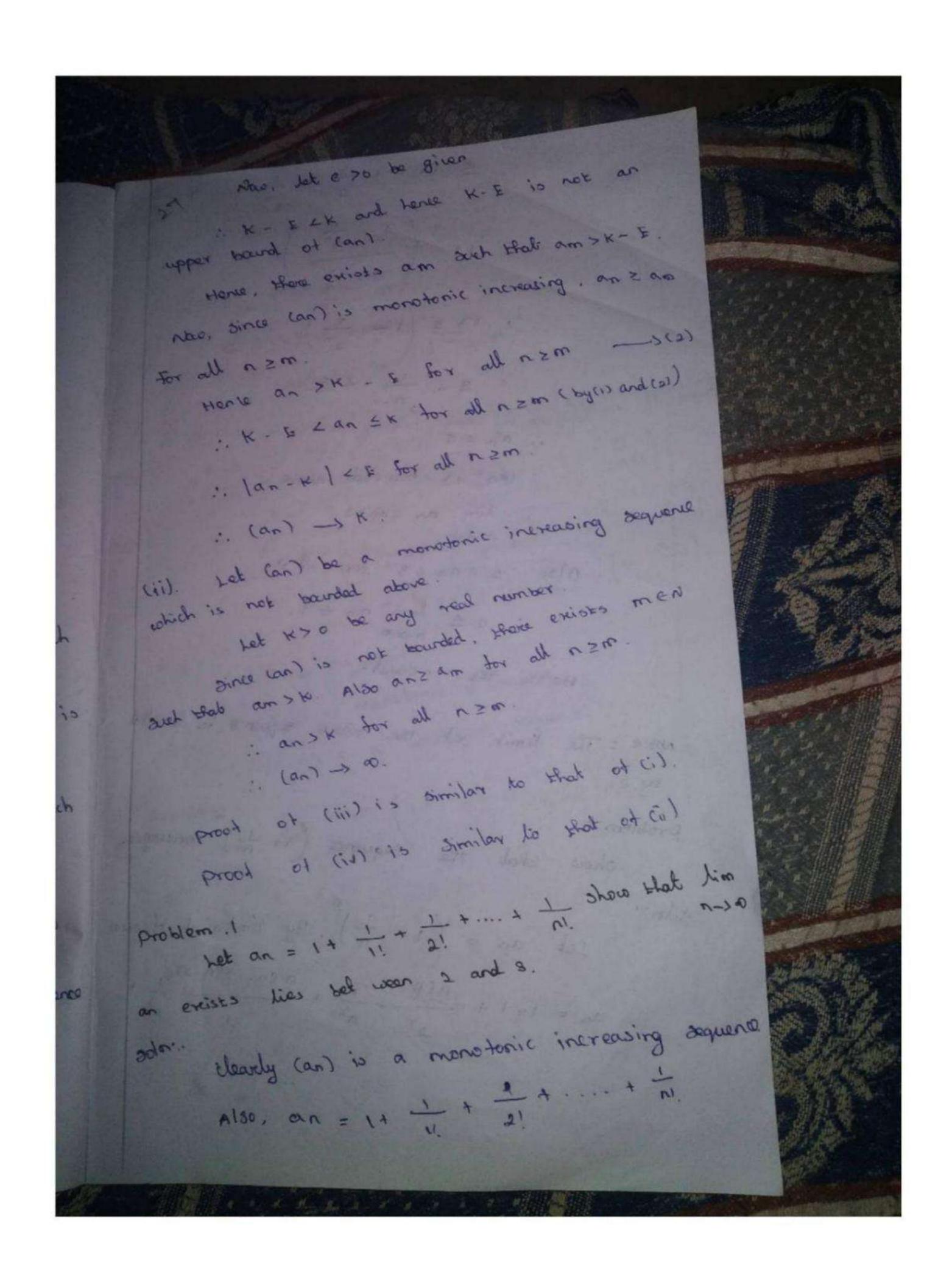


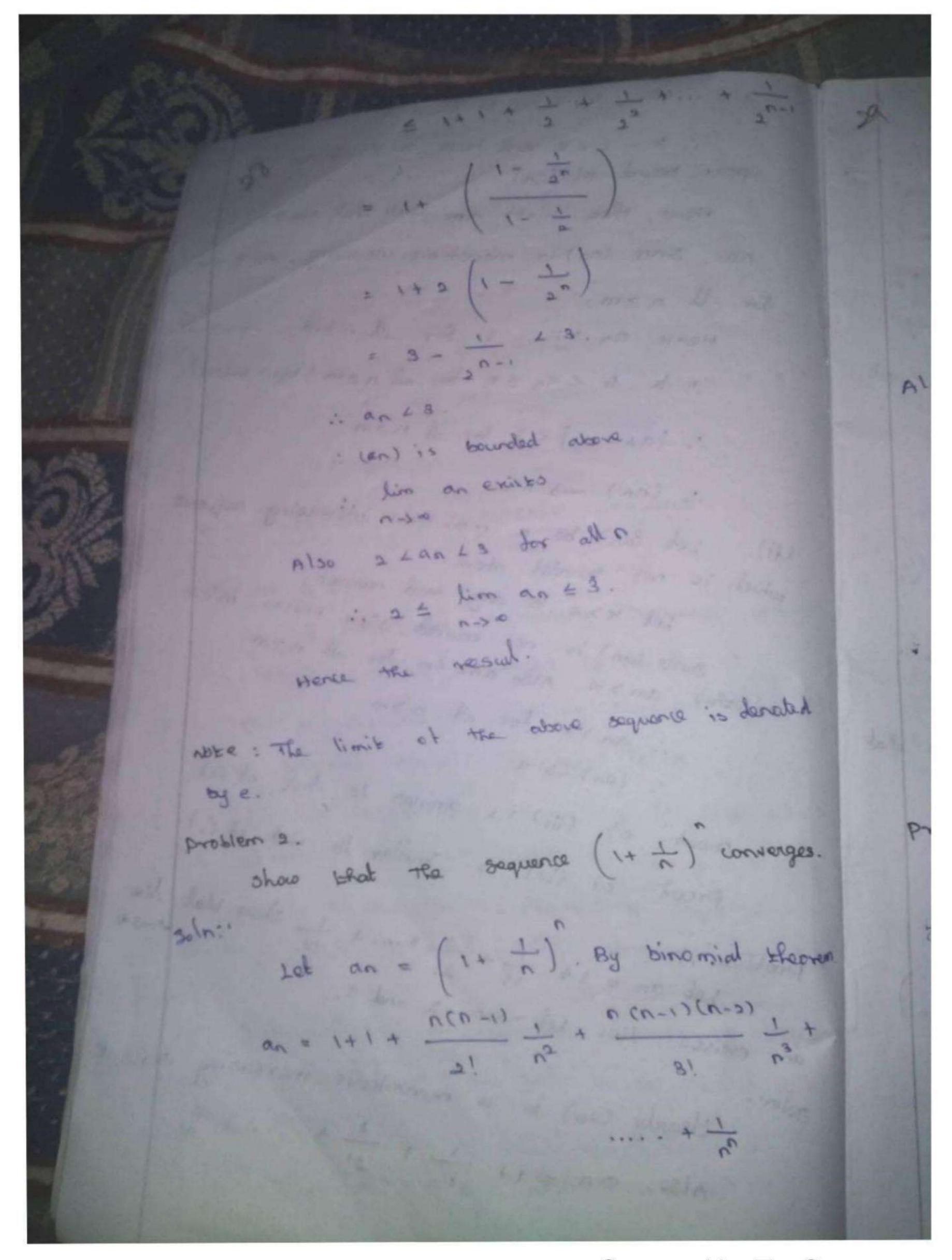


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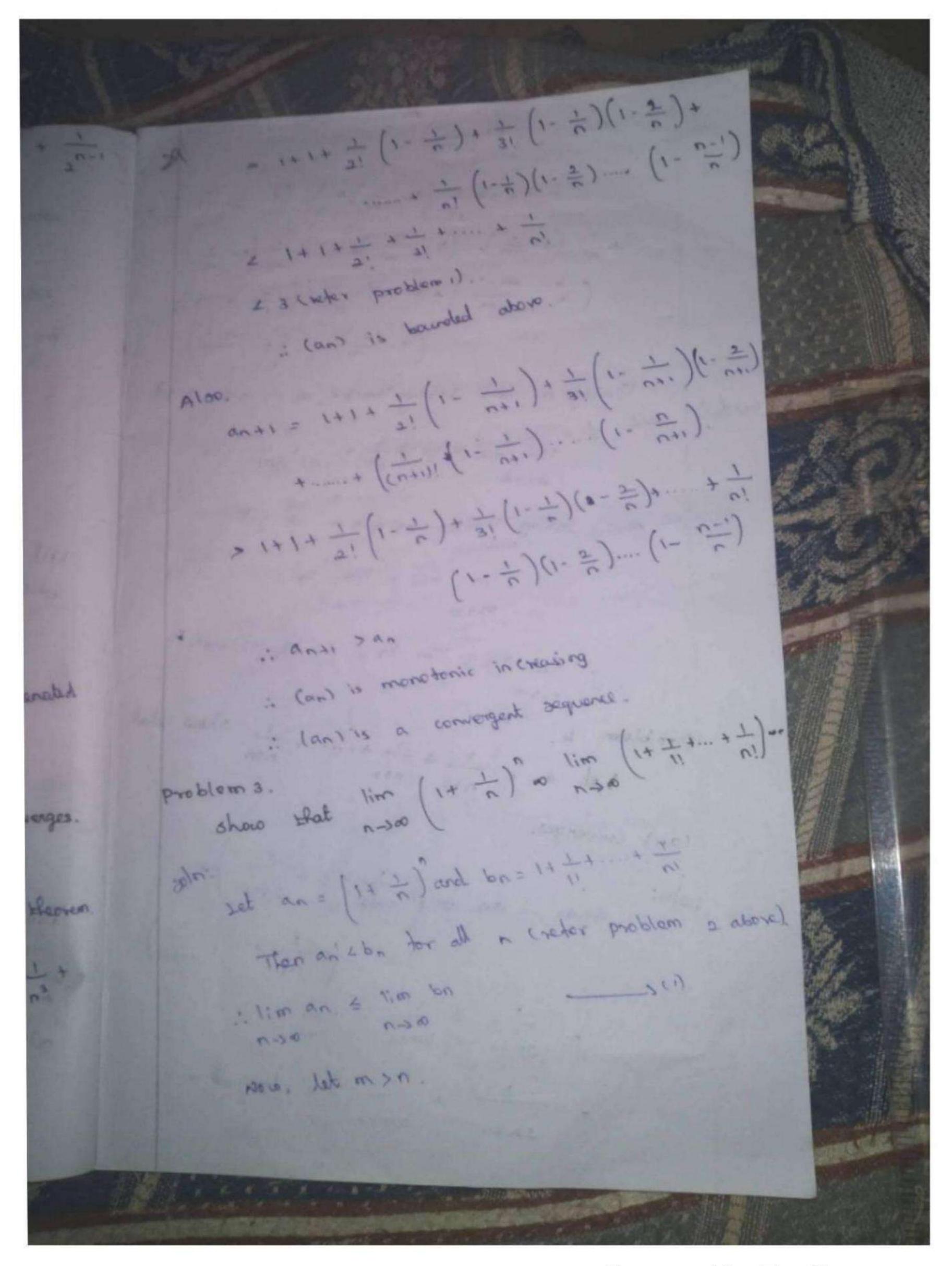




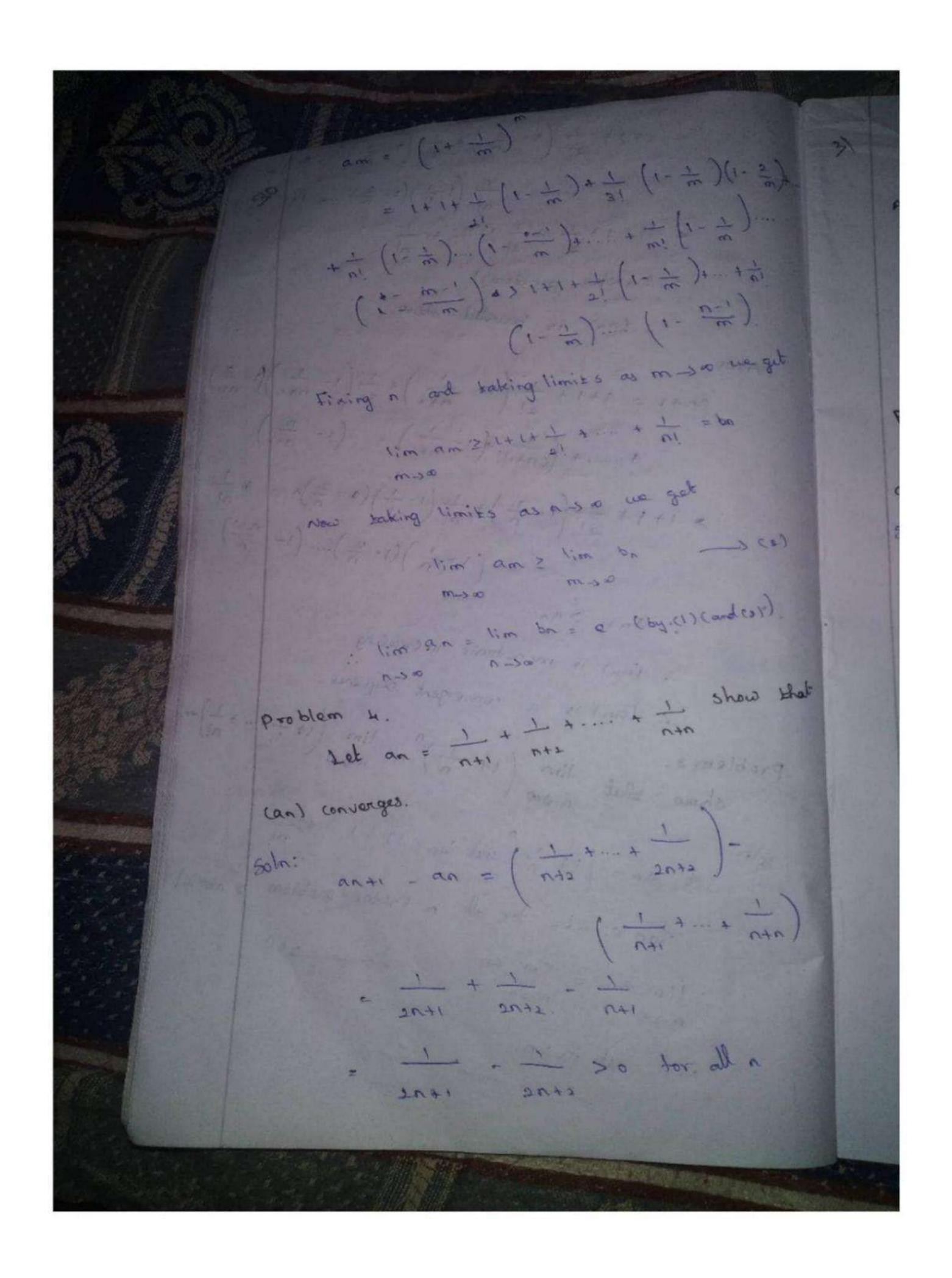




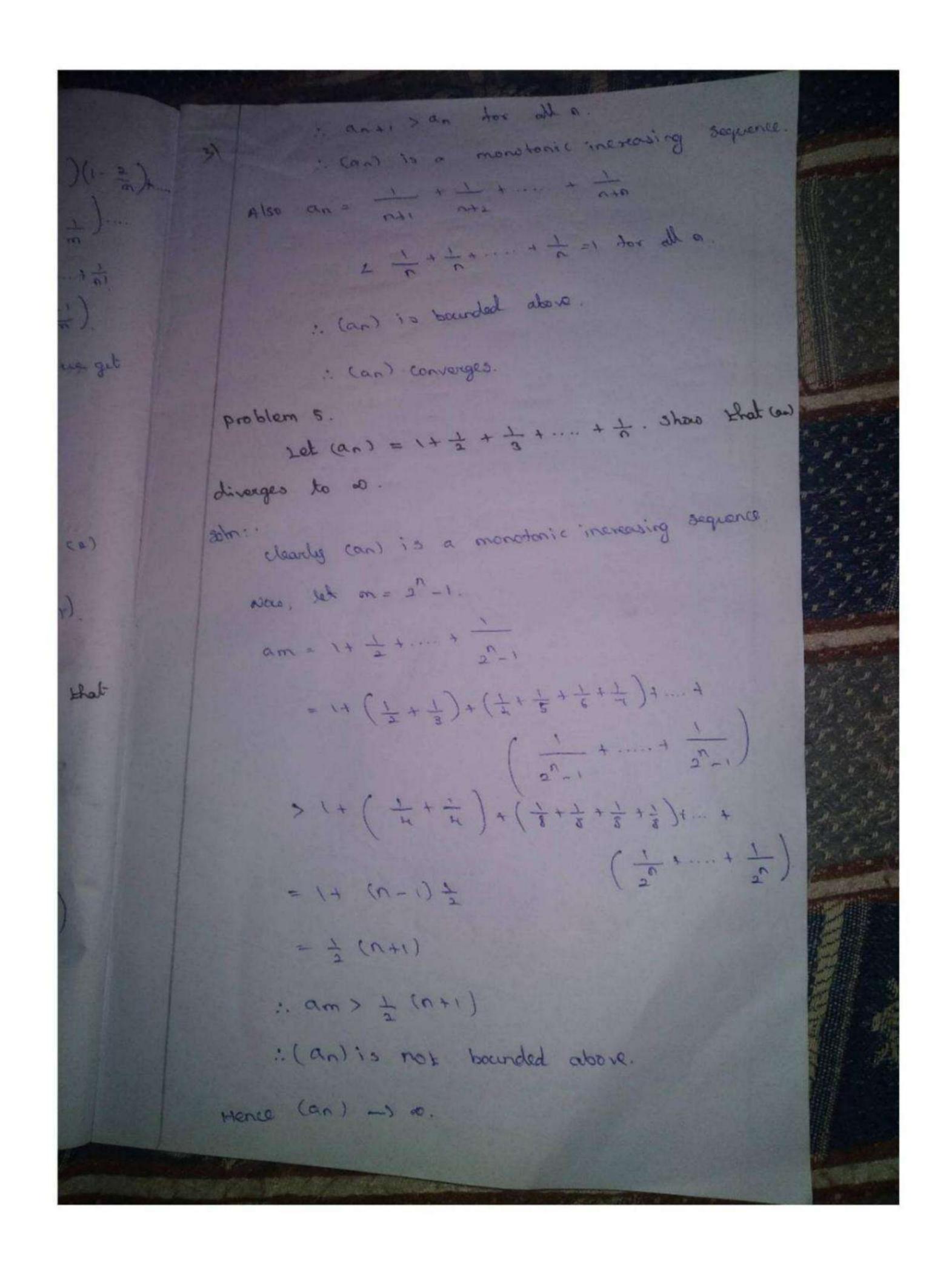
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