Discovery of Stable Isotopes. Using neon gas in his apparatus, Thomson obtained two parab... for the gas itself, a strong one corresponding to a mass 20 and a much weaker one corresponding to a mass 22. The intensity ratio of the two traces was 9 : 1 which gave the relative abundance of the two isotopes. Thomson, therefore, suggested that neon could exist in the form of two *isotopes*, chemically indistinguishable but with different masses 20 and 22. The actual observed atomic weight (20.2) of neon is the weighted mean of the masses of these two isotopes.

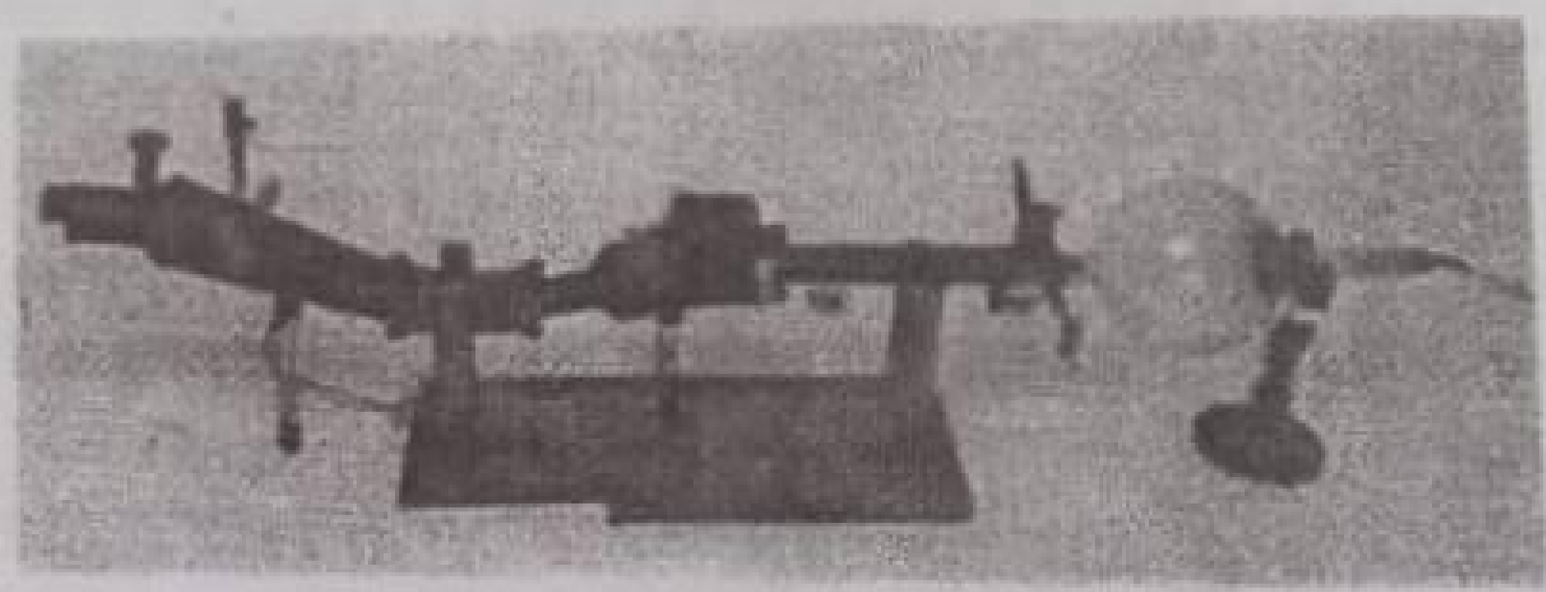
This established the existence of stable isotopes.

Limitations of the Parabola Method:

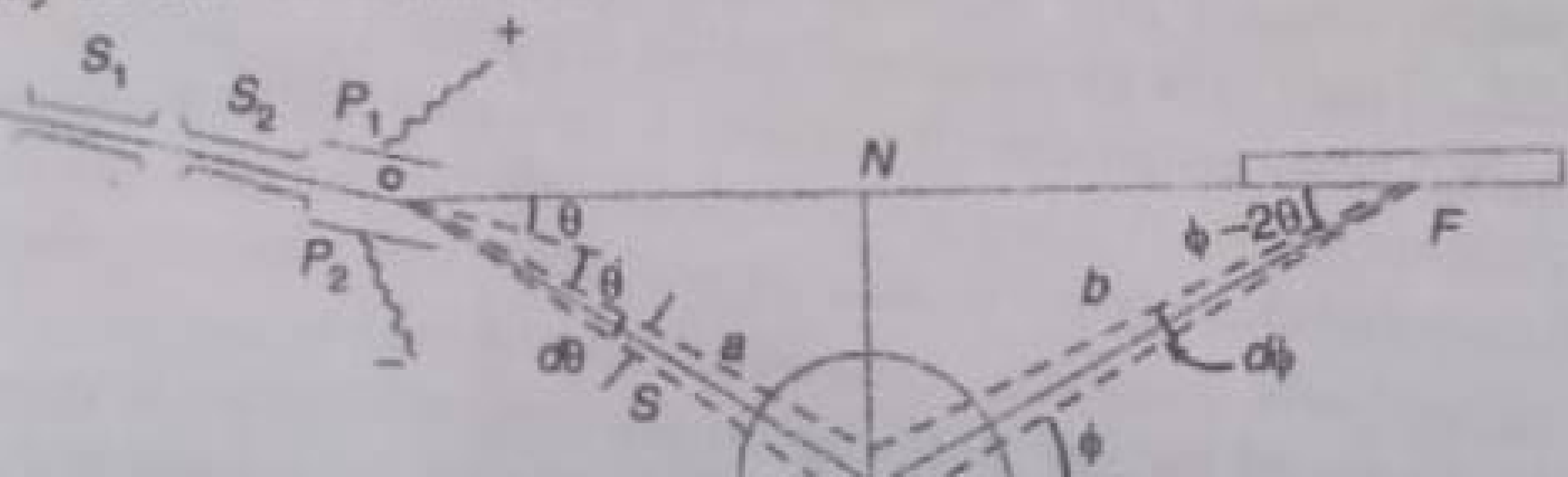
1. Due to the velocity dispersion, each parabolic trace is of very low intensity.
2. The traces on the photographic plate are blurred and have no definite edges. Hence accurate measurements are not possible.
3. The influence of secondary rays makes analysis difficult.

5.4 Aston's Mass Spectrograph

The apparatus used by Aston is shown in Fig. 5.4. The stream of positive ions obtained from a discharge tube is rendered into a fine beam by passing it between two narrow slits S_1 and S_2. This beam enters the electric field between the metal plates P_1 and P_2. Due to the action of the electric field (X), all positive ions having the same value of E/M are not only deviated by an angle θ from the original path but are dispersed by an angle $d\theta$ due to their different velocities. The beam is then allowed to pass through a magnetic field M acting at right angles to the electric field so that it produces a deflection of the beam in the same plane. The magnetic field deviates the particles by an angle ϕ and reconverges them by $d\phi$. The direction and magnitude of the field is so adjusted that it produces a deviation of the beam in the opposite direction and brings all ions having the same value of E/M, eventhough differing in velocities, to a focus at one point F. Ions having different values of E/M are brought to focus at different points on the photographic plate. The condition required for such a focusing may be derived as follows:



Aston's Mass Spectrograph.



Detection of isotopes. To determine the masses of isotopes of an element, the element is mixed with a number of other elements whose atomic masses are known accurately. The traces of all of them are obtained on the same photographic plate. The distances of the traces of the known masses are measured from a given reference point on the plate. A calibration graph is drawn connecting the distances with the atomic masses. By measuring the distances of the traces of the isotopes of the element from the same reference point, their atomic masses are obtained from the calibration graph.

Advantages : 1. All the particles having the same value of E/M are focussed at a single point unlike Thomson's method in which they are spread out into a parabola. Hence, the intensities of the lines on the photographic plate in Aston's mass spectrograph are large, while the parabolic traces on the photographic plate in Thomson's arrangement are feeble.

2. The intensity of a line in the mass spectrum is proportional to the total number of particles of that mass. Hence a rough idea of the relative abundance of various isotopes of an element can be made by this method.

Limitations : 1. The mass scale is not linear.

2. Owing to the polarisation of the electrodes of the electric field, the traces are not quite straight but slightly curved.

Example. *The distance between traces corresponding to masses 12 and 16 in an Aston's mass spectrograph is 4.8 cm. Calculate the mass of the particle whose trace is at a distance of 8.4 cm from the trace of mass 16.*

Sol. Let x be the distance of the fiduciary mark from the trace of mass 16.

$$\therefore \quad x \propto 16. \text{ Also, } x - 4.8 \propto 12.$$

Hence
$$\frac{x}{x-4.8} = \frac{16}{12} \quad \text{or} \quad x = 19.2 \text{ cm.}$$

If M be the mass of the particle whose trace is at a distance of 8.4 cm from the trace of mass 16, we have

$$19.2 \pm 8.4 \propto M \text{ and } 19.2 \propto 16.$$

$$\frac{M}{16} = \frac{19.2 \pm 8.4}{19.2} = 1 \pm \frac{8.4}{19.2} = 1 \pm \frac{7}{16}$$

Hence
$$M = 16 \pm 7 = 23 \text{ or } 9.$$

5.5 Bainbridge's Mass Spectrograph

The apparatus is shown diagrammatically in Fig. 5.5. The beam of positive ions produced in a discharge tube is collimated by two slits S_1 and S_2 and enter a "*velocity selector*". The velocity selector consists of (1) a steady electric field X maintained at right angles to the ion beam between two plane parallel plates P_1 and P_2 and (2) a magnetic field B. The magnetic field is produced by an electromagnet represented by the dotted circle. The magnetic field B is perpendicular to X and the ion beam. The electric field and magnetic field of the velocity selector are so adjusted that the deflection produced by one is nullified by the deflection produced by the other. If X and B are the electric intensity and magnetic induction, then $Xe = Bev$ or $v = X/B$.

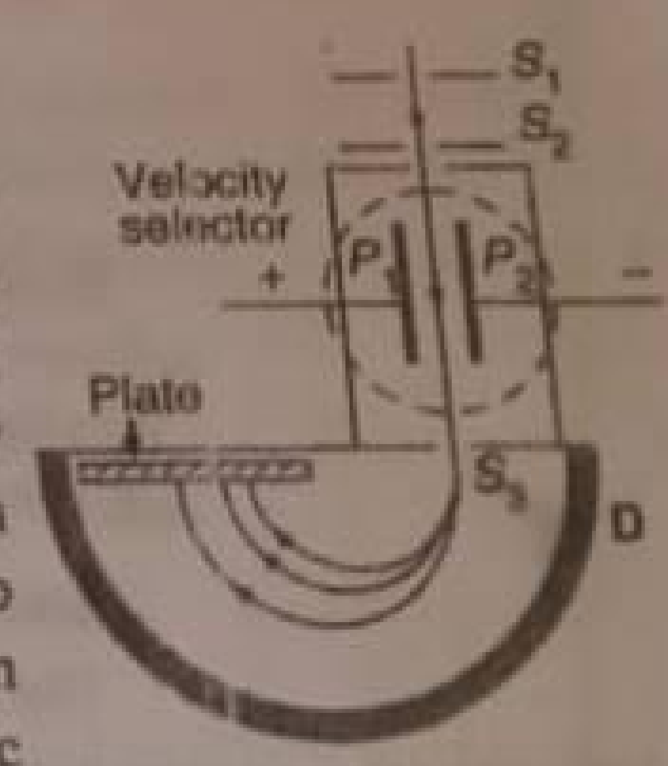


Fig. 5.5

Only those ions, having this velocity v, alone pass through the entry slit S_3 to enter the evacuated chamber D. Thus all ions entering D must have the same velocity. The positive ions which enter into D are subjected to a strong uniform magnetic field of intensity B', perpendicular to its path. The force acting on each ion will be $B'ev$.

Hence,
$$y \propto \frac{BEl^2}{2Mv}$$

or
$$y = k_2 \frac{BE}{Mv} \quad ...(2)$$

Here k_2 is another constant.

Action of Combined Electric and Magnetic Fields.

The combined effect of the two fields is found by eliminating v from (1) and (2).

Squaring (2) and dividing by (1),

$$\frac{y^2}{x} = \left(\frac{k_2^2}{k_1}\frac{B^2}{X}\right)\frac{E}{M} \quad ...(3)$$

B and X are constants. If E/M is constant, then by Eq. (3), $\frac{y^2}{x}$ = constant. This is the equation of a parabola. As Eq. (3) is independent of v, particles of same E/M but of different velocities will fall on different points on the *same parabola.*

$\frac{y}{x} = \frac{k_2}{k_1}\frac{B}{X}v$, *i.e.*, $\frac{y}{x} \propto v$. Thus, the position of any individual particle on the parabola will depend on the velocity of the particle.

The ions having different values of E/M will lie along the different parabolas.

For one direction of the magnetic field, one half of the parabola is traced. Reversing the magnetic field, the other half is also traced. When the full parabola is traced, it is easy to draw the axis of symmetry (X-axis).

Determination of E/M. The value of E/M can be calculated from Eq. (3) by measuring the coordinates x and y for a point on the parabola, evaluating the constants k_1 and k_2 for the apparatus and knowing B and X.

Determination of Mass. The mass of a positive ion is determined in terms of the mass M_1 of the standard hydrogen ion. A small trace of hydrogen is always present in all samples of gases. Hydrogen, being the lightest element, gives the outermost parabola. Let I and II represent the parabolic traces due to ions of the gas of mass M_2 and the hydrogen ions of mass M_1 respectively (Fig. 5.3). An ordinate a,b,c,d,e is drawn. It cuts the two parabolas at a, b, d, e and the X-axis at c. Let ac and bc represent the two values of y corresponding to a constant value of x on these parabolas. Let us assume that both ions have the same charge. Then from Eq. (3),

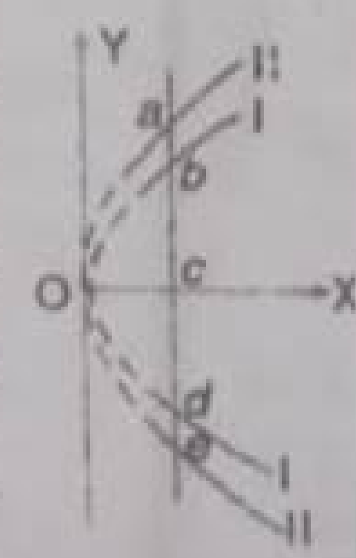


Fig. 5.3

$$\frac{ac^2}{x} = \left(\frac{k_2^2}{k_1}\frac{B^2}{X}\right)\frac{E}{M_1}$$

and
$$\frac{bc^2}{x} = \left(\frac{k_2^2}{k_1}\frac{B^2}{X}\right)\frac{E}{M_2}$$

or
$$\frac{M_2}{M_1} = \frac{ac^2}{bc^2} = \left(\frac{ae}{bd}\right)^2$$

The lengths ae and bd can be measured on the photograph. Hence the parabolic traces enable us to compare the masses of different ions with hydrogen used as a standard.

$$M_2 = \left(\frac{ae}{bd}\right)^2 M_1.$$

Now for the same drop under the same electric field (X), mg/Xv is constant. Hence $v_2 - v_1 \propto (n_2 - n_1)e$ *i.e.*, any change in the velocity of the drop is proportional to the quantity of the acquired charge. Millikan made thousands of observations on a single drop. He found that there was a minimum value of $(v_2 - v_1)$ and that other values of $(v_2 - v_1)$ were simple integral multiples of this $(v_2 - v_1)_{minm}$. $(v_2 - v_1)_{minm}$ corresponds to the addition or subtraction of one unit of charge. *i.e.*, $(n_2 - n_1) = 1$ when $(v_2 - v_1)$ is minimum.

$$\therefore \quad e = \pm\frac{mg}{Xv}(v_2 - v_1)_{minm} \quad ...(8)$$

Substituting the value of m from (4),

$$e = \pm\frac{4}{3}\pi\left(\frac{9\eta}{2}\right)^{\frac{3}{2}}\left[\frac{v}{(\rho-\sigma)g}\right]^{\frac{1}{2}}\frac{(v_2-v_1)_{minm}}{X} \quad ...(9)$$

The value of electronic charge e found by Millikan was 1.591×10^{-19} C. The value of e now generally accepted is 1.602×10^{-19} C.

Balanced Oil-Drop Method. In this method, the intensity of the electric field is adjusted to a suitable value X such that the drop remains at rest. Then the effective weight of the drop (mg) exactly balances the force due to the electric field (Xe).

$\therefore$ $Xe = mg$ or $e = mg/X$. (We have assumed that the drop carries one charge).

Substituting the value of m from (4),

$$e = \frac{4}{3}\frac{\pi}{X}\left(\frac{9\eta v}{2}\right)^{\frac{3}{2}}\frac{1}{[(\rho-\sigma)g]^{1/2}}$$

4.2 The Free Electron Theory of Metals

Electrical Conduction in Metals. The following assumptions are made in the electron theory:

(*i*) In *conductors* there is a large number of *free electrons*, actually separated from the atoms. They are capable of moving freely in the interatomic spaces.

(*ii*) These electrons (conduction electrons) are treated as an *electron gas*. The electrons are performing random motion with all possible velocities like the gas molecules. The mean K.E. of the electrons is equal to that of the gas molecules at the same temperature. The positive ions in the metal occupy fixed positions.

In the absence of an electric field, the free electrons move about irregularly in the whole volume of the metal like the gas molecules. When an electric field is applied between the ends of a conductor, there will be an electron drift in a direction opposite to the direction of the electric field, since the electrons carry a negative charge. The effect of the electric field is to impart a velocity called "drift

Mass of Mg^{24} nucleus $= 24 \times (1.67 \times 10^{-27})$ kg.

$$\therefore \quad R_1 = (3.75 \times 10^{24}) \times (24 \times 1.67 \times 10^{-27}) = 0.150 \text{ m.}$$
$$R_2 = (3.75 \times 10^{24}) \times (25 \times 1.67 \times 10^{-27}) = 0.156 \text{ m.}$$
$$R_3 = (3.75 \times 10^{24}) \times (26 \times 1.67 \times 10^{-27}) = 0.163 \text{ m.}$$

Example 3. *In a Bainbridge mass spectrograph, singly ionised atoms of Ne^{20} pass into the deflection chamber with a velocity of 10^5 ms^{-1}. If they are deflected by a magnetic field of flux density 0.08 T, calculate the radius of their path and where Ne^{22} ions would fall if they had the same initial velocity.*

Sol. For an ion of Ne^{20} moving in a magnetic field B we can write

$$Bev = \frac{Mv^2}{R_{20}} \text{ or } R_{20} = \frac{Mv}{Be}$$

Here, $M = 20/(6.023 \times 10^{26})$ kg; $v = 10^5$ ms^{-1}, $B = 0.08\,T$ and $e = 1.602 \times 10^{-19}$ C.

$$\therefore \quad R_{20} = \frac{(20/6 \times 10^{26}) \times 10^5}{0.08 \times (1.602 \times 10^{-19})} = 0.26 \text{ m}$$

If e is the same for all ions, then $R \propto M$. The radius of the path followed by Ne^{22} is given by

$$\frac{R_{22}}{R_{20}} = \frac{22}{20} \text{ or } R_{22} = \frac{22}{20} \times R_{20} = \frac{22}{20} \times 0.26 = 0.286 \text{ m.}$$

$$2R_{22} - 2R_{20} = 2 \times 0.026 \text{ m} = 0.052 \text{ m.}$$

Hence Ne^{22} ions would fall 5.2 cm away from the Ne^{20} ions.

5 mark

5.6 Dempster's Mass Spectrograph

The experimental arrangement is shown in Fig. 5.6. The anode is a metal cylinder A with its front surface C coated with a salt of the element under test and heated electrically. A filament, if electrically heated by the battery B_1, emits electrons. By maintaining the filament at a P.D. of about 53 volts with respect to A by using another battery B_2, the electrons are made to bombard the heated salt with the result that the anode emits positively charged ions of the element. These ions are collimated into a narrow beam by the slit S_1. Then the positive ions are accelerated towards the slit S_2 by a variable P.D. V maintained between S_1 and S_2.

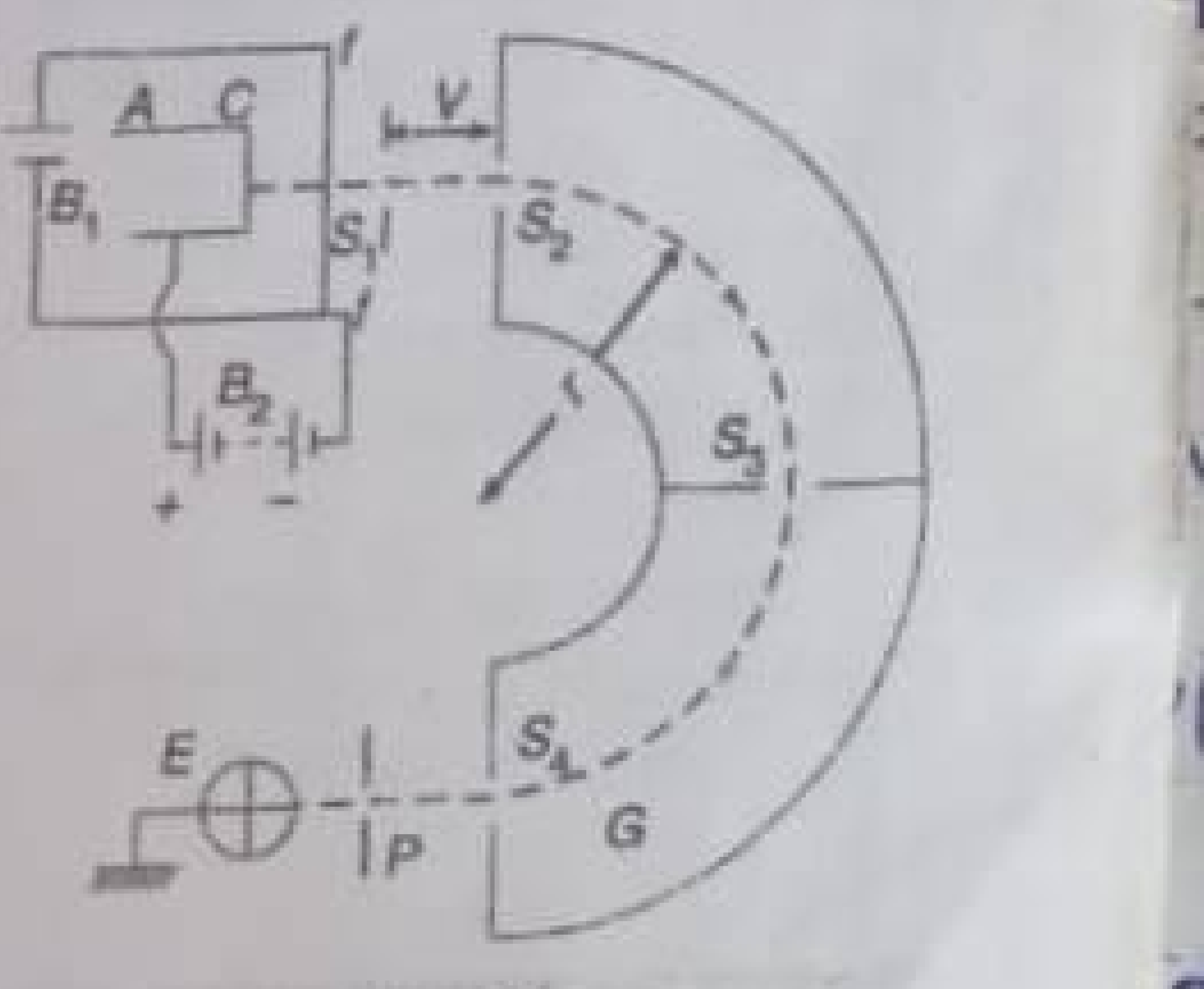


Fig. 5.6

We know that when ions of mass M and charge e are accelerated through a P.D. V, they acquire a velocity v given by

$$\frac{1}{2} Mv^2 = eV$$

v is the velocity of the ions

or $$v = \sqrt{\frac{2eV}{M}} \qquad \ldots(1)$$

Ions with different masses trace circular paths of different radii given by

$$R = \frac{Mv}{B'e}\left(\because B'ev = \frac{Mv^2}{R}\right)$$

$$\therefore \quad \frac{e}{M} = \frac{v}{B'R} = \frac{X}{BB'R}\left(\because v = \frac{X}{B}\right)$$

Since v and B' are constant quantities, $\frac{e}{M} \propto \frac{1}{R}$.

After describing semicircles, the ions strike a photographic plate.

Now, $M = \frac{B'eR}{v}$. If e is the same for all ions, then $M \propto R$.

So we get a linear mass scale on the photographic plate. It will be seen that ions of different masses strike the photographic plate at different points, thus giving a typical mass spectrum.

Advantages : (1) Since a linear mass scale is obtained, accuracy of measurements is increased.

(2) The sensitivity depends on the strength of the deflecting magnetic field B' and the field area of the chamber D. Bainbridge used a magnetic field of 1.5 weber/m^2 over a semicircle of radius 0.2m. He found a definite increase in resolving power over Aston's apparatus. The ten isotopes of tin were resolved by this instrument.

Example 1. *A beam of positively charged particles go through a velocity selector where an electric field of 6×10^5 volt/metre and a magnetic induction of 2×10^{-2} weber/m^2 are applied perpendicular to each other. Then the particles go through another magnetic induction of 1.25 weber/m^2 applied perpendicular to the path. If the diameter of the path taken is one metre, calculate the specific charge of the particles.*

Sol. We have, $\frac{e}{M} = \frac{X}{BB'R}$. Here, $X = 6 \times 10^5$ volt/metre;

$B = 2 \times 10^{-2}$ weber/m^2, $B' = 1.25$ weber/m^2 and $R = 0.5$ m.

$$\therefore \quad \frac{e}{M} = \frac{6 \times 10^5}{(2 \times 10^{-2})(1.25)\,0.5} = 4.8 \times 10^7 \text{ coulomb/kg.}$$

Example 2. *Singly-ionised Mg atoms enter a Bainbridge mass spectrograph with the velocity selector having electric and magnetic fields respectively of 30 kilovolts/metre and 0.1 tesla. Calculate the radii of the path followed by the three isotopes of masses 24, 25 and 26 when the deflecting magnetic field is 0.5 tesla.*

Sol. The mass of a nucleon is 1.67×10^{-27} kg.

The ions emerging from the velocity selector have a velocity given by

$$v = \frac{X}{B} = \frac{30000}{0.1} = 3 \times 10^5 \text{ ms}^{-1}$$

Now, if B' is the "deflecting" magnetic field, the radius of the path of an ion of mass M, velocity v and charge e is given by

$$R = \frac{Mv}{B'e} = \frac{3 \times 10^5 M}{(0.5)(1.6 \times 10^{-19})} = 3.75 \times 10^{24} M.$$

Let R_1, R_2 and R_3 be the radii of isotopes of mass numbers 24, 25 and 26 respectively.

Experimental Procedure. The plates are first connected together, so that the electric field between them is zero. A single drop is observed through the microscope as it descends under the action of gravity. As the drop is moving in a viscous medium (air), it will move with a constant terminal velocity v. Using a stop watch, the terminal velocity of the drop is measured. Let

v = terminal velocity of the free fall,
a = radius of the drop,
ρ = density of oil,
σ = density of air,
η = coefficient of viscosity of air,
m = effective mass of the drop and
g = acceleration due to gravity.

Then by Stokes law, $mg = 6\,\pi\,\eta a v$ ...(1)

$$\text{or } \frac{4}{3}\pi a^3\,(\rho-\sigma)g = 6\,\pi\eta a v \qquad ...(2)$$

$$\text{or} \qquad a = \left[\frac{9\eta v}{2(\rho-\sigma)g}\right]^{\frac{1}{2}} \qquad ...(3)$$

$$\therefore \text{ The effective mass of the drop} = m = \frac{4}{3}\pi a^3\,(\rho-\sigma) = \frac{4}{3}\pi\left[\frac{9\eta v}{2(\rho-\sigma)g}\right]^{\frac{3}{2}}(\rho-\sigma) \qquad ...(4)$$

Thus, by measuring v, we can calculate the effective mass m of the chosen oil drop.

Now, an electrostatic field X is applied steadily, so that the drop moves upwards with a steady velocity v_1. If the total charge on the drop is $n_1\,e$ (where n_1 is an integer),

the resultant upward force on the drop $= Xn_1e - mg$.

According to Stokes law, $Xn_1e - mg = 6\,\pi\eta\,av_1$ ...(5)

Dividing (5) by (1), $\dfrac{Xn_1\,e - mg}{mg} = \dfrac{v_1}{v}$ or $\dfrac{Xn_1\,e}{mg} = \dfrac{v_1+v}{v}$

$$\therefore \quad n_1\,e = \frac{mg\,(v_1+v)}{Xv} \qquad ...(6)$$

Now, the air between the plates is irradiated with X–rays. X–rays ionize the air molecules. The oil drop under observation may pick up one or more ionic charges. Due to the acquisition of charges, the velocity of the drop changes suddenly, when X is constant. Let the new velocity of the drop be v_2 and the corresponding charge on the drop $n_2\,e$. Then,

$$n_2\,e = \frac{mg\,(v_2+v)}{Xv} \qquad ...(7)$$

$$\therefore \quad (n_2-n_1)e = \frac{mg}{Xv}\,(v_2-v_1)$$

If $v_2 > v_1$, $(n_2-n_1)\,e$ is positive and if $v_2 < v_1$, $(n_2-n_1)\,e$ is negative. Both are possible, as both positive and negative ions are present.

4 THE ELECTRON AND BAND THEORY OF SOLIDS

AT A GLANCE			
4.1	Determination of the Electronic Charge : Millikan's Oil-drop Method	4.2	The Free Electron Theory of Metals
4.3	Expression for Electrical Conductivity	4.4	Expression for Thermal Conductivity
4.5	Electron Microscope	4.6	Band Theory of Solids
4.7.	Classification of Solids on the Basis of Band Theory	4.8	Optical Properties of Solids
4.9	Energy Bands: Alternative Analysis		

10 Mark Question

4.1 Determination of the Electronic Charge : Millikan's oil-drop Method

Experimental arrangement. The apparatus consists of two optically plane parallel brass plates *A* and *B* (Fig. 4.1), separated by insulating rods of glass. The lower plate *B* is earthed. The upper plate *A* can be charged to a positive potential of the order of 10000 volts, from a high-tension battery. This observation chamber is situated in a bigger chamber *C* which is completely surrounded by a constant temperature bath of oil *E*. Tiny drops of a heavy non-volatile oil are sprayed into the chamber by an atomizer *D*, Some of the oil drops enter the space between the plates through the pinhole *H* in the top plate *A*. These drops are charged, due to the frictional effect at the nozzle of the atomizer. The air between the plates can be ionized by allowing X-rays to pass through it. Then the drops may pick up additional charges. The drops are illuminated by light from an arc lamp *S*. The oil-drop is observed by means of a short-focus telescope *T* provided with a millimetre scale in the eyepiece.

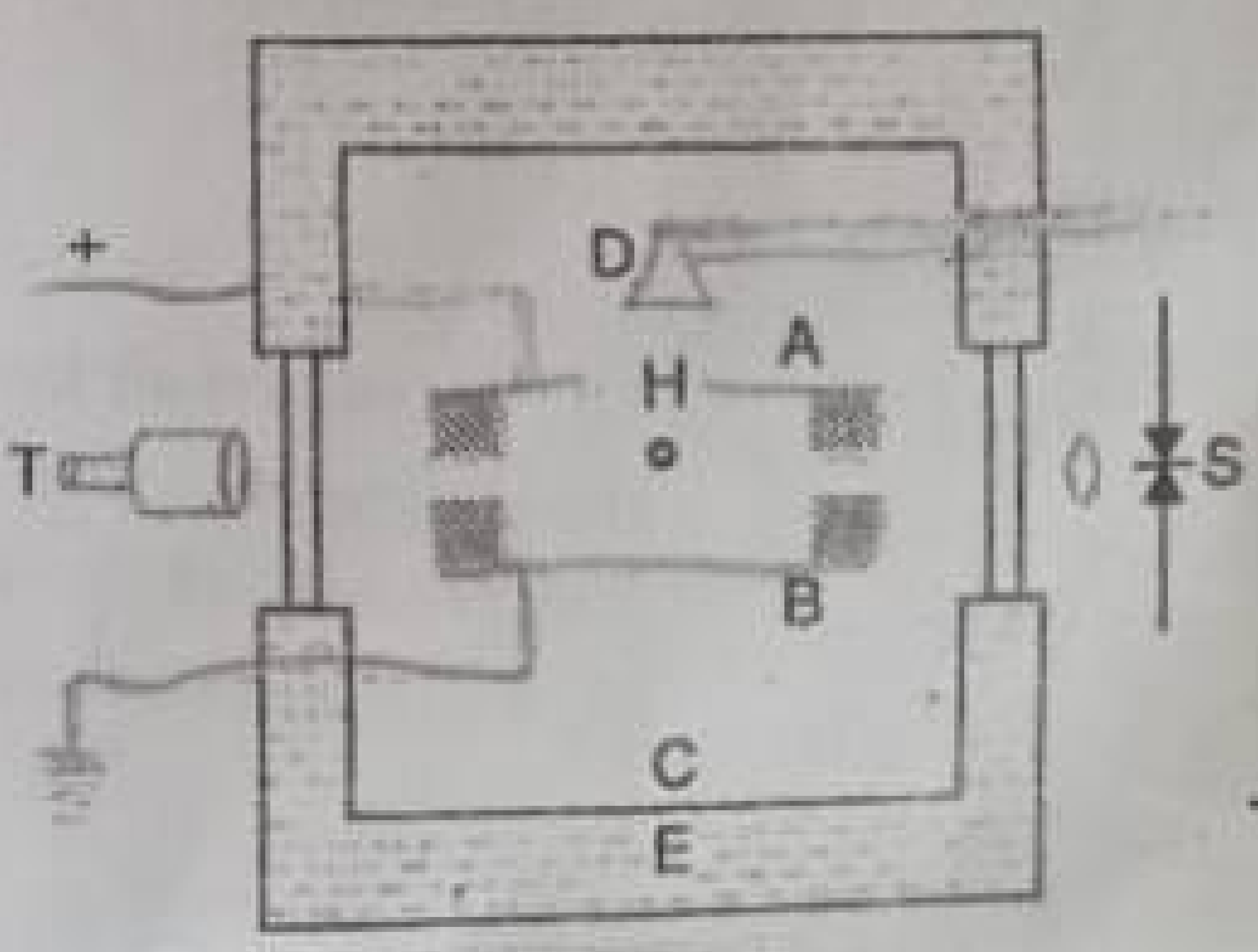


Fig. 4.1

different glancing angles. The graph of ionisation current against glancing angle is drawn. The graph obtained is as in Fig. 5.7 and is called an ***X-ray spectrum.***

- The prominent peaks A_1, A_2, A_3 refer to X-rays of wavelength λ. The glancing angles $\theta_1, \theta_2, \theta_3$ corresponding to the peaks A_1, A_2, A_3 are obtained from the graph. It is found that

  $$\sin \theta_1 : \sin \theta_2 : \sin \theta_3 = 1 : 2 : 3.$$

- This shows that A_1, A_2, A_3, refer to the first, second and third order reflections of the same wavelength. B_1, B_2, B_3 are such peaks for the first, second and third order for another wavelength (λ_2). Thus Bragg experimentally verified the relation

  $$2d \sin \theta = n\lambda.$$

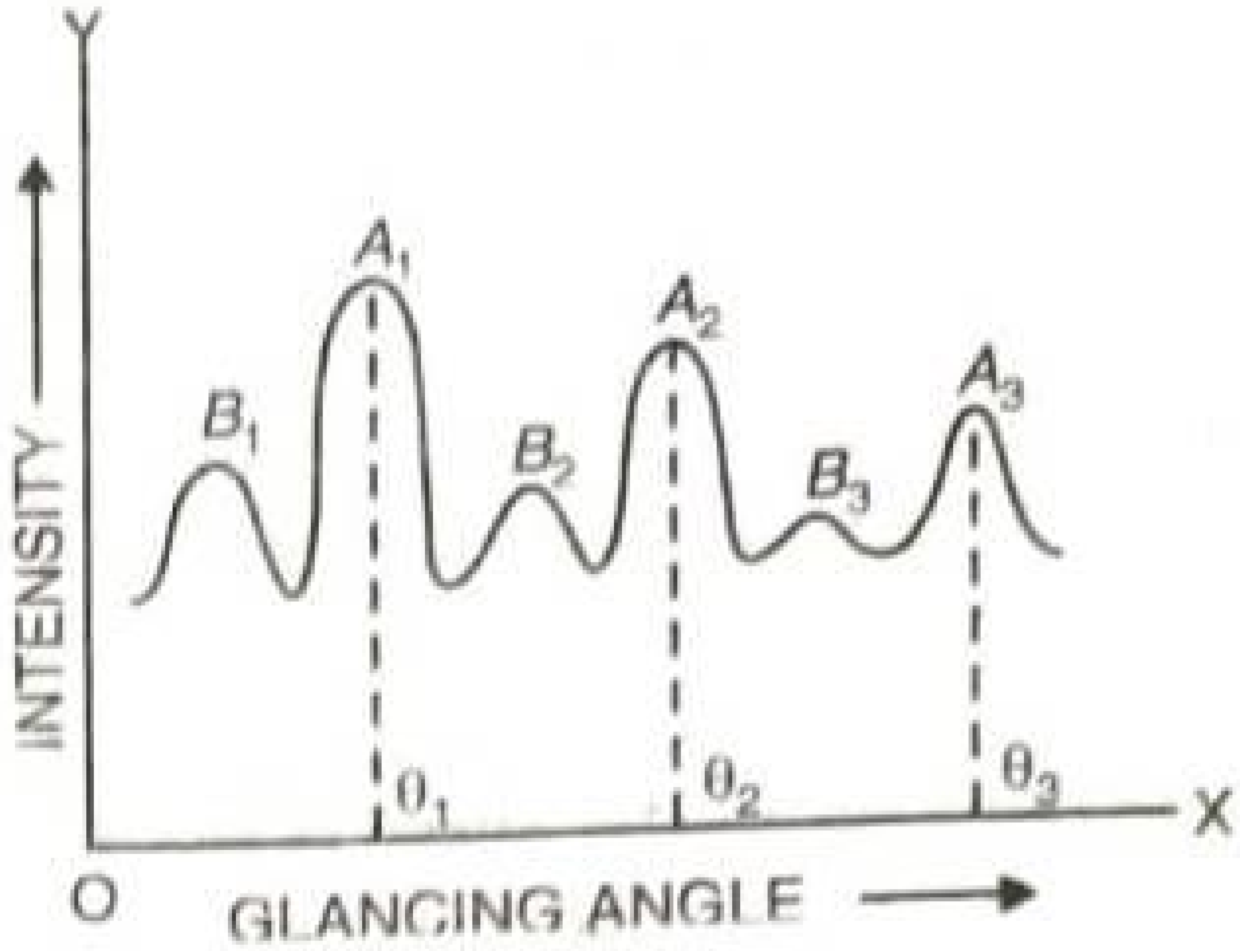


Fig. 5.7

- The wavelength of X-rays is determined by using the equation $2d \sin \theta = n\lambda$. The glancing angle θ is experimentally determined as explained already for a known order. If d is known, λ can be calculated.

Calculation of *d*. Rocksalt (*NaCl*) possesses a cubic structure with sodium ions and chlorine ions situated alternately at corners of a cube. If d is the distance between two neighbouring ions and ρ is the density of the crystal, then mass of the unit cube $= \rho d^3$. Now each corner ion is shared by 8 neighbour cubes. This is illustrated by the ion indicated by an asterisk in Fig. 5.8. Hence each ion contributes only $\frac{1}{8}$th of its mass to the cube. The unit cube is made up of 4 sodium ions and 4 chlorine ions. Hence, the mass of each cube = the mass of $\frac{1}{2}$ *NaCl* molecule. If M is the molecular weight of *NaCl* and N_A the Avogadro's No.,

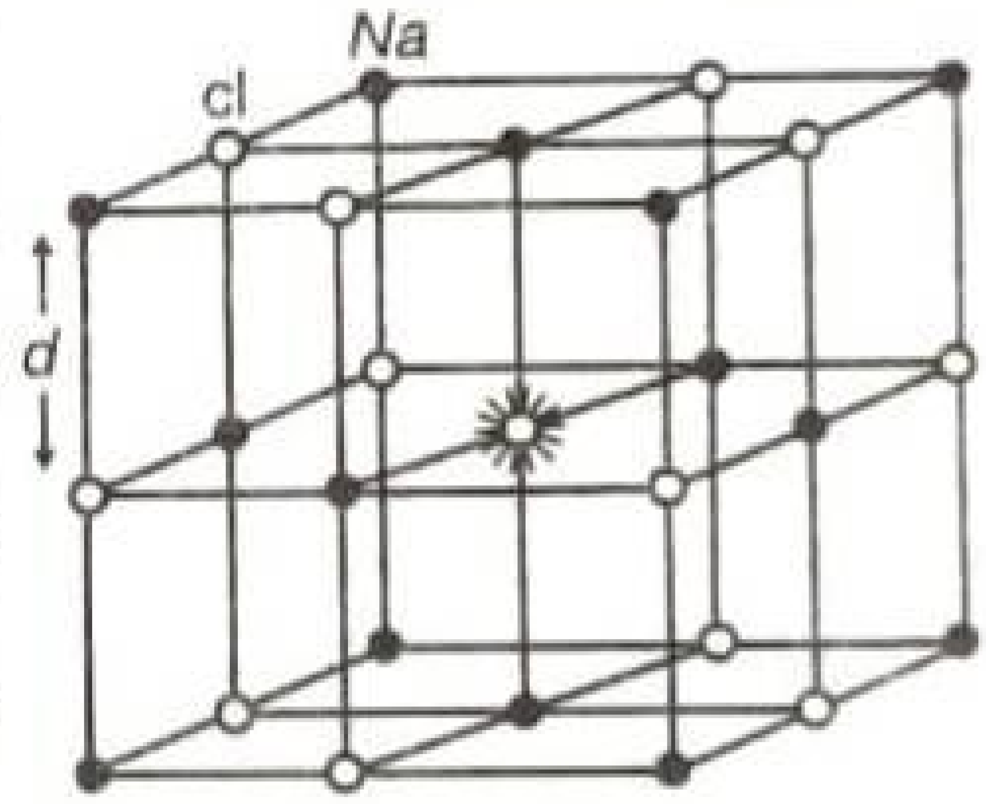


Fig. 5.8

Mass of $\frac{1}{2}$ *NaCl* molecule $= M/2N_A$

$$\therefore \qquad \rho d^3 = \frac{M}{2N_A} \text{ or } d = 3\sqrt{\frac{M}{2N_A \rho}}$$

Knowing M, N_A and ρ, d can be found.

For *NaCl*, $M = 58.45$ kg ; $\rho = 2170$ kg m^{-3} and $N_A = 6.06 \times 10^{26}$.

$$\therefore \qquad d = 3\sqrt{\frac{58.45}{2 \times (6.06 \times 10^{26}) \times 2170}} = 2.81 \times 10^{-10} \text{ m.}$$

EXAMPLE 1. *The spacing between principal planes of NaCl crystal is* 2.82 Å. *It is found that first order Bragg reflection occurs at an angle of* 10°. *What is the wavelength of X–rays ?*

SOL. By Bragg equation, $2d \sin \theta = n\lambda$.

Here, $d = 2.82 \times 10^{-10}$m; $n = 1$ and $\theta = 10°$. $\lambda = ?$

$$\lambda = \frac{2d \sin \theta}{n} = \frac{2 \times (2.82 \times 10^{-10}) \sin 10°}{1} = 0.98 \times 10^{-10} \text{ m.}$$

Explanation, according to Bohr's theory. Bohr's theory of hydrogen spectrum gives the frequency of a spectral line as

$$\nu = Z^2 Rc\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

Here, R is Rydberg's constant and c the velocity of light.

K_α line originates from the transition of electron from second to first orbit.

Now, $n_1 = 1$ and $n_2 = 2$.

$$\therefore \quad \text{frequency of } K_\alpha \text{ line} = \nu = Z^2 Rc\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = \frac{3}{4}cRZ^2$$

This approximately corresponds to Moseley's law.

Importance of Moseley's law. (1) According to this law, it is the atomic number and not atomic weight of an element which determines its characteristic properties, both physical and chemical. Therefore, the atoms must be arranged in the periodic table according to their atomic numbers and not according to their atomic weights. This would remove some discrepancies in the order of certain elements from the point of view of their atomic weights. For example, argon $_{18}Ar^{40}$ comes before potassium ($_{19}K^{39}$), cobalt ($_{27}Co^{58.9}$) comes before nickel ($_{28}Ni^{58.7}$), etc. So the arrangement is correct in the order of atomic number.

(2) Moseley's work has also helped to perfect the periodic table by (*i*) the discovery of new elements, *e.g.*, hafnium (72), illinium (61), masurium (43), rhenium (75), etc., and (*ii*) the determination of the atomic numbers of rare-earths and fixing their positions in the periodic table.

5.10 POLARISATION OF X-RAYS

Experimental arrangement. Fig. 5.14 shows the experimental arrangement for showing the polarisation of X-rays. A beam of X-rays travelling along AZ is allowed to strike at a scattering sheet Z. This scatters X-rays in all directions. With the help of slits S_3 and S_4, a beam in the direction ZO which is perpendicular to the direction OX is isolated. The beam ZO strikes another scatterer at O. An ionization chamber $I.C.$ is placed to receive X-rays in the directions OX and OY. It is observed that the intensity of X-rays scattered in the direction OX is maximum and is nearly zero in the OY direction. The beam ZO is said to be polarized.

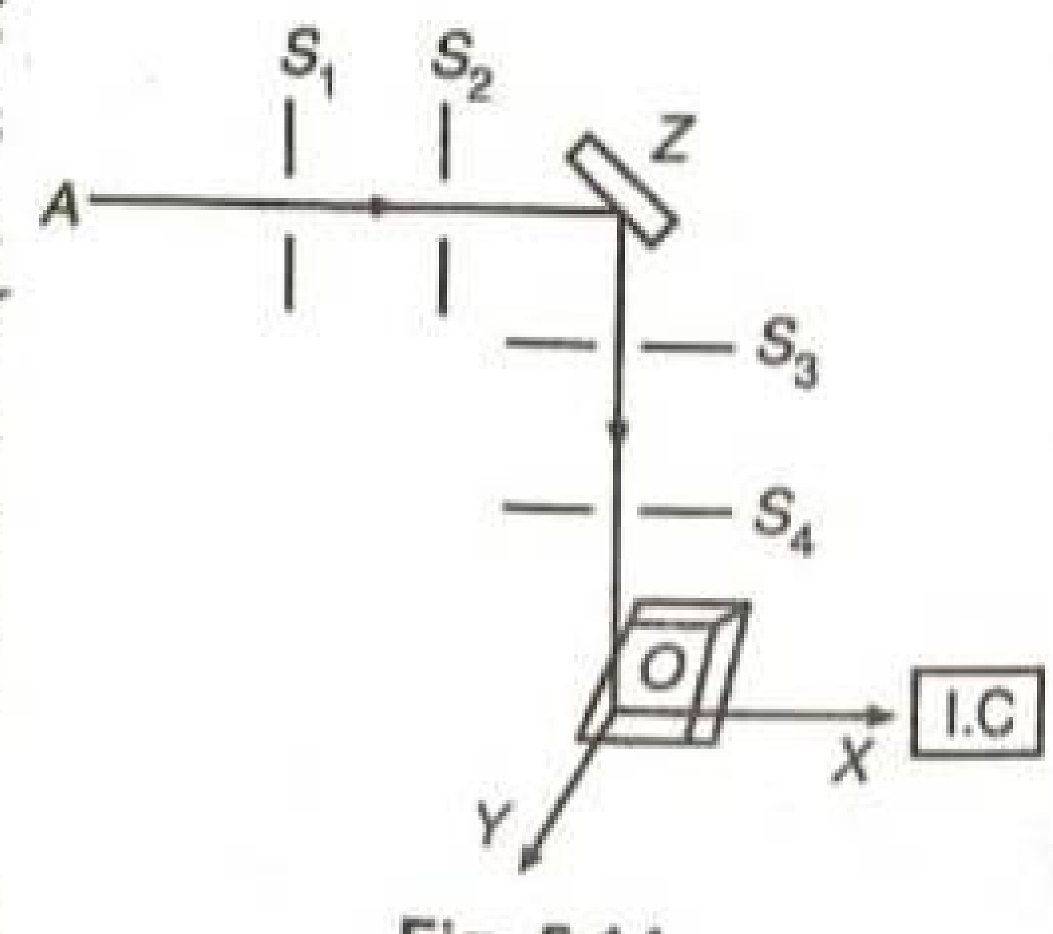


Fig. 5.14

Thus X-rays from the X-ray tube become completely polarized after having been scattered in the direction at right angles to the initial direction. The scatterer at Z behaves like a polarizer and that at O as an analyser.

5.11 SCATTERING OF X-RAYS (THOMSON'S FORMULA)

When an electromagnetic wave interacts with a single free electron, the electron performs simple harmonic motion and in so doing reradiates. The radiations thus emitted by the vibrating electron under the influence of the incident X-ray beam are called *scattered X-rays*.

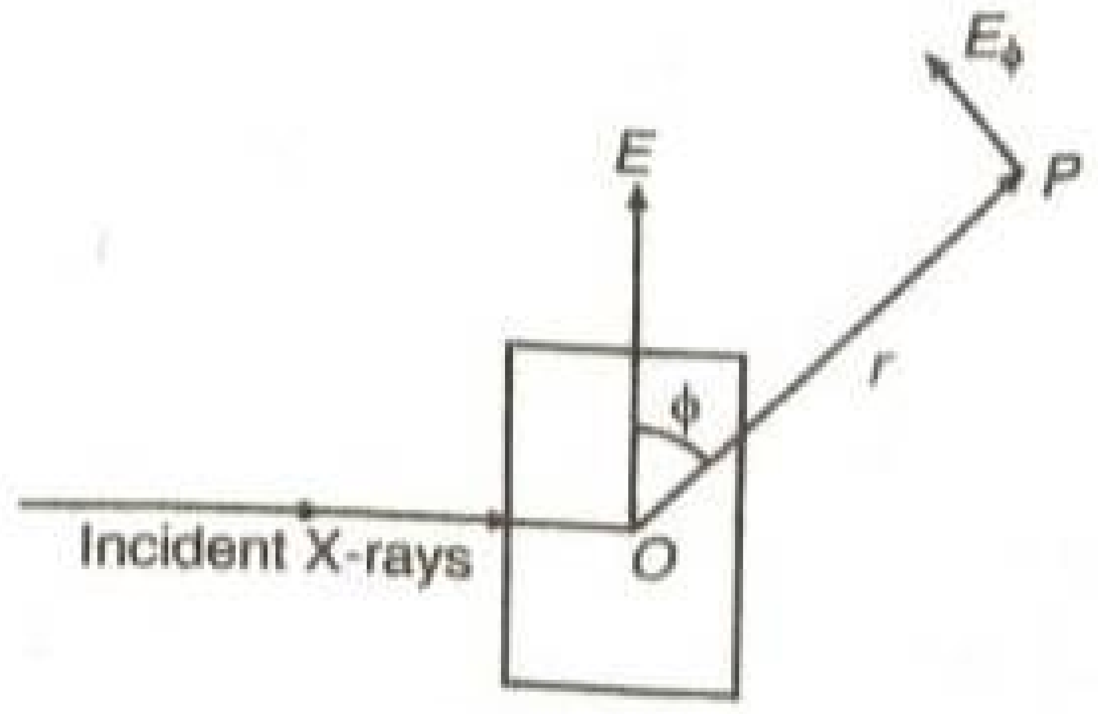


Fig. 5.15

At any point O within the material let E and B be the electric and magnetic vectors of the incident X-ray beam (Fig. 5.15). Both are perpendicular to the incident direction of the beam. The electron at the point O will experience an straight line, the value of h/e is obtained. Then substituting the known value of e, h is calculated. The value of h calculated in this way agrees fairly well with the value obtained by other methods. Thus the Einstein's equation can be verified experimentally.

6.6 PHOTOELECTRIC CELLS

Photoelectric cell is an arrangement to convert light energy into electrical energy. There are three types of photocells, photoemissive, photovoltaic and photoconductive.

(i) Photo-emissive Cell.

Construction. This consists of a glass or quartz bulb (B) according as it is to be used with visible or ultraviolet light (Fig. 6.11). C is the silver cathode in the form of a semi-cylindrical plate. The anode (A) is a rod mounted vertically at the centre of the bulb and parallel to its axis. A positive potential of about 100 volts is applied to the anode, the negative being connected to the cathode through a galvanometer G.

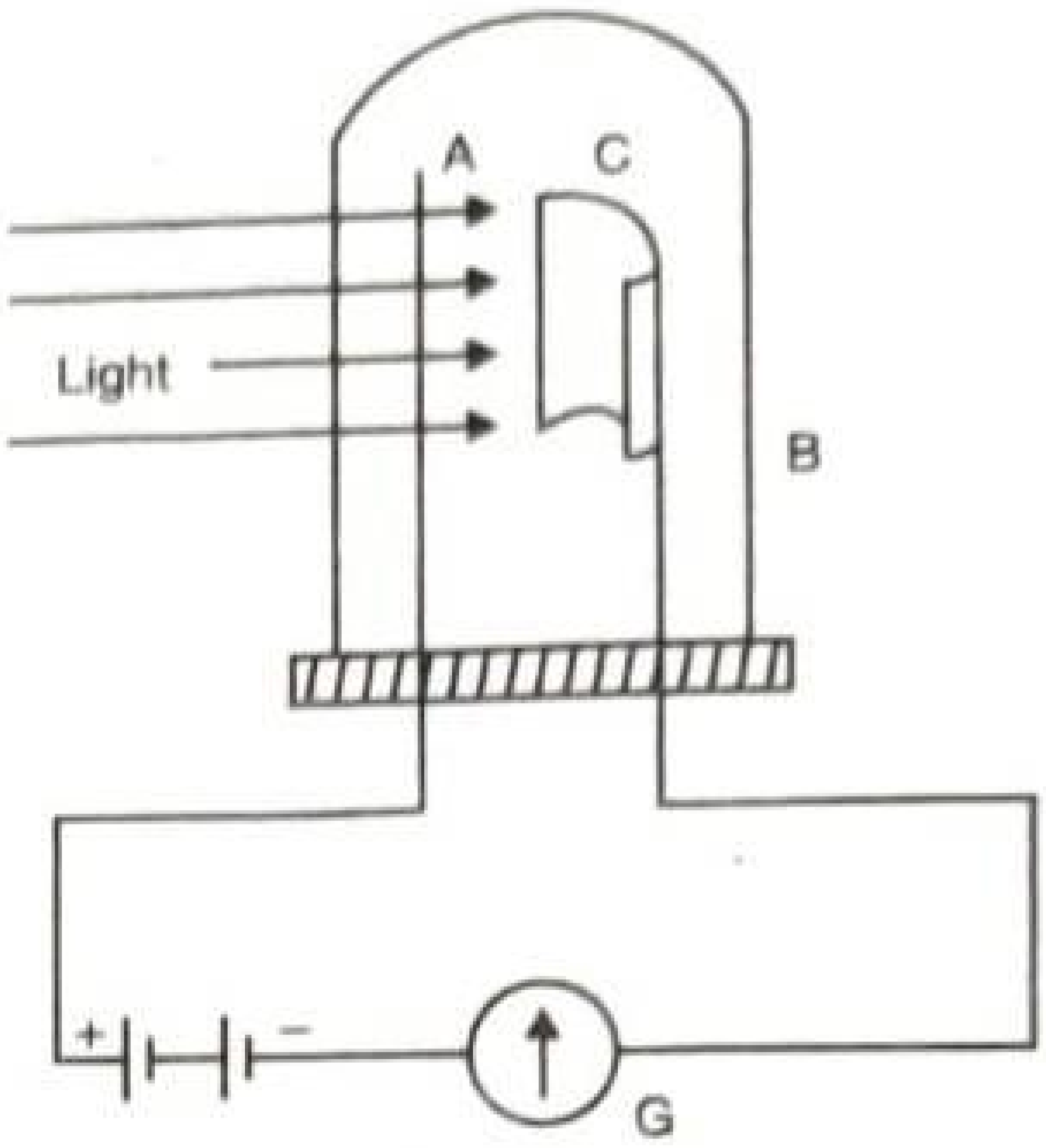


Fig. 6.11

Working. When light falls on the cathode C, electrons are ejected from the cathode. A small current flows through the cell and can be measured by the galvanometer.

- The photoemissive cell is used for reproduction of sound from photo-films.

(ii) Photo-voltaic Cell.

Construction. It consists of a layer of semiconductor material spread over a metallic surface by heat treatment. The metal plate is made of copper. The semiconductor is cuprous oxide (Cu_2O).

On the other side of the semiconductor, there is a very thin layer of a translucent deposit which allows the semiconductor to be illuminated by radiations (Fig. 6.12).

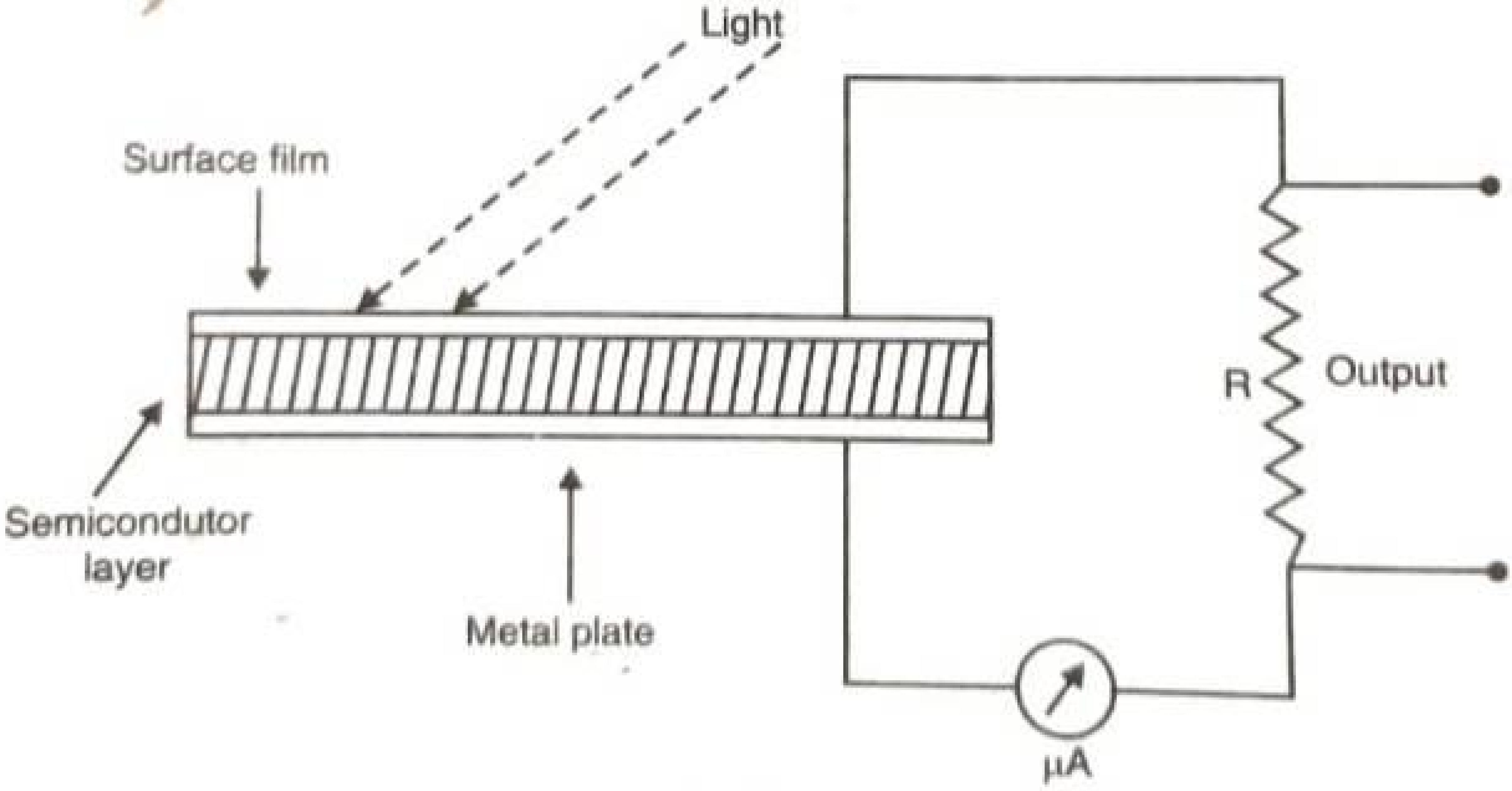


Fig. 6.12

Working. Light falling on the surface film (of gold or silver) penetrates into it and ejects photoelectrons from the semiconductor layer. These electrons travel in a direction opposite to the direction of the incident light. The conventional current, therefore, flows in the direction of the incident light. For small values of the resistance of the galvanometer, the current is directly the same for all emitting surfaces and is found to be equal to h. The value of ν_0 however, depends on the emiting surface. ν_0 is called the threshold frequency, as it represents the beginning of the photoelectric activity of the emitter.

Definition. *Threshold frequency is defined as the minimum value of frequency of incident light below which the photo-electric emission stops completely, howsoever high the intensity of light may be.*

- At threshold frequency, the K.E. of emitted photo-electrons is just zero.

6.4 EXPERIMENTAL INVESTIGATIONS ON THE PHOTOELECTRIC EFFECT

Apparatus: Photoelectric effect can be studied in detail with the help of the apparatus shown in Fig. 6.8.

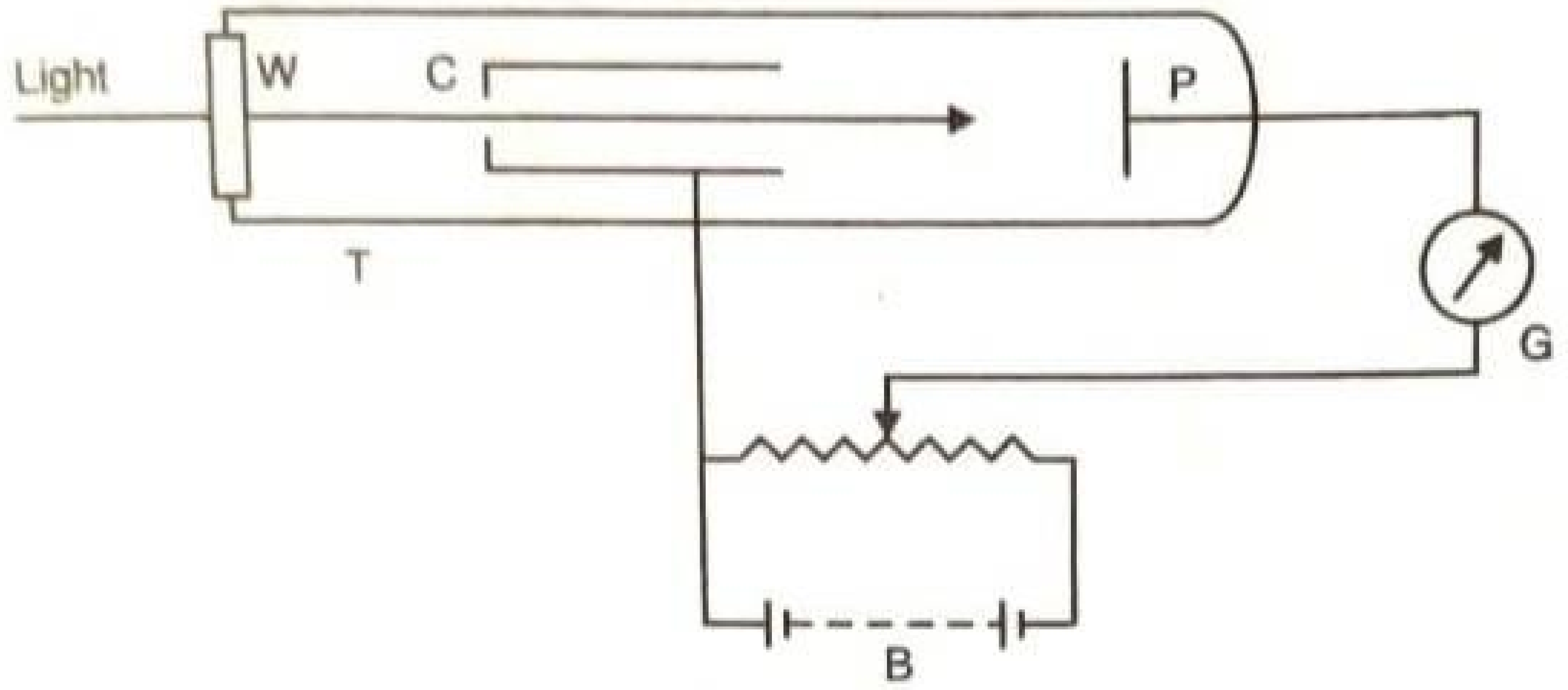


Fig. 6.8

It consists of an evacuated glass tube T with a quartz window W. P is a photoelectrically sensitive plate. C is a hollow cylinder and it has a small hole that permits the incident light to fall on the plate P. P is connected to the negative end. C is connected to the positive terminal of a battery B.

Working. When light from some source falls on the plate P, the photoelectrons are ejected out of the plate P. These photoelectrons are attracted by the positively charged cylinder C. Hence a photoelectric current flows from P to C in the bulb and from C to P outside the bulb. This current can be measured from the deflection produced in the galvanometer G. It is found that the strength of the photoelectric current increases as the potential of C is more and more positive with respect to P. The deflection in G decreases when the potential of C is negative with respect to P. The results obtained are summarised into four statements. They are called the laws of photoelectric emission.

Laws of photoelectric emission. (*i*) *For every metal, there is a particular minimum frequency of the incident light, below which there is no photoelectric emission, whatever be the intensity of the radiation. This minimum frequency, which can cause photoelectric emission, is called the threshold frequency.*

(*ii*) *The strength of the photoelectric current is directly proportional to the intensity of the incident light, provided the frequency is greater than the threshold frequency.*

(*iii*) *The velocity and hence the energy of the emitted photoelectrons is independent of the intensity of light and depends only on the frequency of the incident light and the nature of the metal.*

(*iv*) *Photoelectric emission is an instantaneous process. The time lag, if any, between incidence of radiation and emission of the electrons, is never more than 3×10^{-9} sec.*

Failure of the electromagnetic theory. The above experimental facts could not be explained on the basis of the electromagnetic theory of light.

(1) Calculations showed that it would require about 500 days to dislodge a photoelectron from sodium by exposure to violet light of wavelength 4000 Å. Experimentally, however, we observe that electron ejection commences without delay.

or
$$V = \frac{h}{e}\nu - \frac{\phi}{e} \qquad ...(1)$$

ϕ is constant for a given metal; h and e are also constants.

Hence, Eq. (1) represents a straight line. V is measured for different values of ν. A graph is then plotted between the stopping potential (V) taken along the Y-axis and the frequency of light (ν) taken along the X-axis. The graph is a straight line (Fig. 6.9). The slope of the straight line

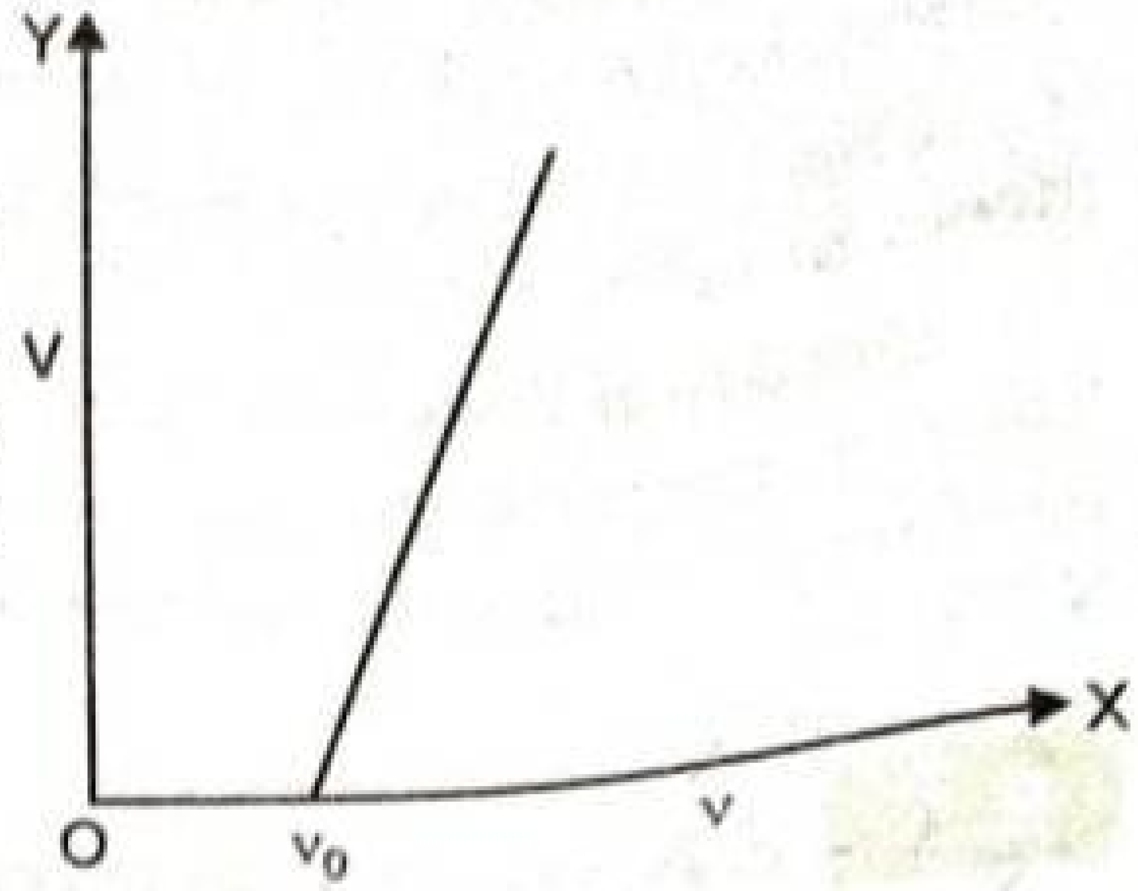


Fig. 6.9

$$\tan\theta = \frac{h}{e}$$

$$\therefore \qquad h = e\tan\theta \qquad ...(2)$$

Hence the value of h (Planck's constant) can be calculated. The intercept on the X-axis gives the threshold frequency ν_o for the given emitter. From this, photoelectric work function $= \phi = h\ \nu_o$ is calculated.

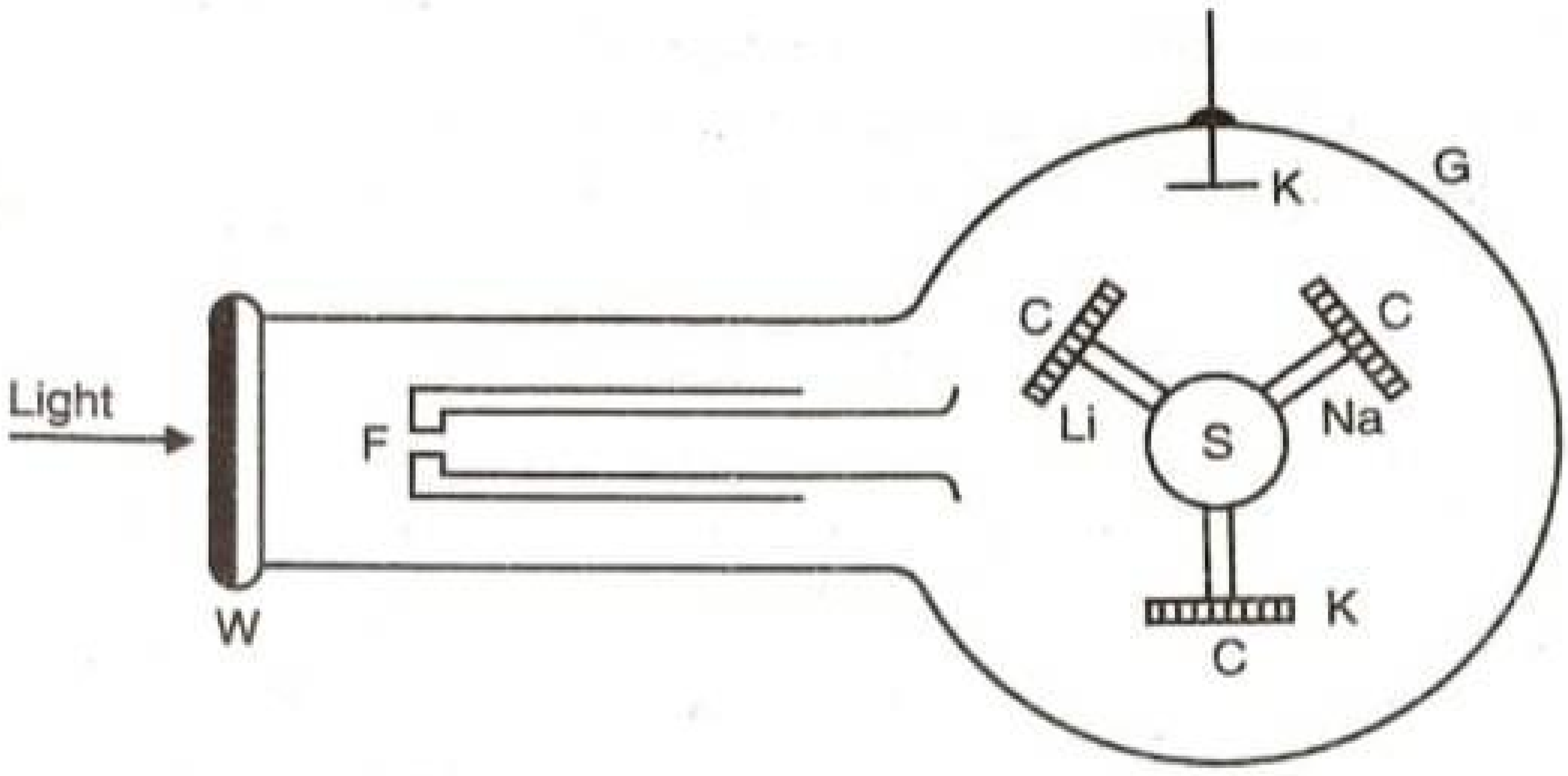


Fig. 6.10

Experiment. Millikan's apparatus is shown in Fig. 6.10. Alkali metals are employed as emitters, since they readily exhibit photoelectric emission even with visible light. Cylindrical blocks (C) of sodium, potassium or lithium are mounted on a spindle S at the centre of the glass flask G. The flask is evacuated to a very high vacuum to free the metals from all absorbed gases and to prevent their oxidation. The spindle can be rotated from outside by an electromagnet. As each metal block passes by the adjustable sharp edge K, a thin layer of it is removed, thus exposing a fresh surface of the metal to the irradiating light entering the flask through a quartz window W. Monochromatic light provided by a spectroscope is used to illuminate the fresh metal surfaces. The photoelectrons are collected by a Faraday cylinder F. The Faraday cylinder is made of copper oxide which is not photosensitive. The photocurrent is measured by an electrometer connected to the Faraday cylinder.

Robert Andrews Millikan (1868–1953)

The stopping potential of the liberated photoelectrons is measured by raising the emitter surface to a positive potential, just sufficient to prevent any of the electrons from reaching the collector (F). The stopping potential is the positive potential applied to the emitter, which corresponds to zero current in the electrometer. The stopping potential (V) is determined for different wavelengths of the incident light. The value of V should be corrected for any contact potential between the metal (C) and Faraday cylinder (F). On plotting V against ν, we get a straight line. Measuring the slope of the

(2) According to the classical theory, light of greater intensity should impart greater K.E. to the liberated electrons. But, this does not happen. Also, the velocity of the emitted electron should not depend on the frequency of the incident light. But it does.

The phenomenon was adequately explained by Einstein on the basis of Planck's Quantum theory of radiation.

Quantum theory. According to Planck, the energy of a monochromatic wave with frequency ν can only assume those values which are integral multiples of energy $h\nu$, *i.e.*, $E_n = nh\nu$, where n is an integer referring to the number of "*Photons*". Thus the energy of a single *PHOTON* of frequency ν is $E = h\nu$.

6.5 EINSTEIN'S PHOTOELECTRIC EQUATION

According to Einstein, light of frequency ν consists of a shower of corpuscles or photons each of energy $h\nu$. When a photon of light of frequency ν is incident on a metal, the energy is completely transferred to a free electron in the metal. A part of the energy acquired by the electron is used to pull out the electron from the surface of the metal and the rest of it is utilised in imparting K.E. to the emitted electron. Let ϕ be the energy spent in extracting the electron from the emitter to which it is bound (photoelectric work function) and $\frac{1}{2}mv^2$ the K.E. of the photoelectron.

Then
$$h\nu = \phi + \frac{1}{2}mv^2 \qquad \text{...(1)}$$

This relation is called the *Einstein's Photoelectric equation.* If ν_0 is the threshold frequency which just ejects an electron from the metal without any velocity then, $\phi = h\nu_0$.

$\therefore$
$$h\nu = h\nu_0 + \frac{1}{2}mv_{max}^2 \qquad \text{...(2)}$$

Here, v_{max} is the maximum velocity acquired by the electron.

or
$$\frac{1}{2}mv_{max}^2 = h(\nu - \nu_0) \qquad \text{...(3)}$$

- *The work function of a metal is defined as the energy which is just sufficient to liberate electrons from the metal surface with zero velocity.*
- Equation (3) suggests that the energy of the emitted photoelectrons is independent of the intensity of the incident radiation but increases with the frequency.

Experimental verification of Einstein's Photoelectric Equation—Millikan's Experiment.

Theory. Millikan's experiment is based on the "*stopping potential*". The stopping potential is the necessary retarding potential difference required in order to just halt the most energetic photoelectron emitted.

The K.E. of a photoelectron leaving the surface of a metal irradiated with light of frequency ν $\Big\} = \frac{1}{2}mv_{max}^2 = h\nu - \phi$.

Let V be the P.D. which is applied between the emitter and a collecting electrode in order to prevent the photoelectron from just leaving the emitter, the emitter being maintained at a positive potential with respect to the collector. Then,

$$eV = \frac{1}{2}mv_{max}^2$$

$\therefore$
$$eV = h\nu - \phi$$

(i) **Relation between photoelectric current and retarding potential.** They first studied the ralation between the photoelectric current I and the retarding potential V. Irradiating the cathode C with monochromatic light of a given intensity, the cathode potential (V) was varied from a few volts positive upto zero and negative values. The photoelectric current I was measured for different values of V. The observations were repeated with the intensity of illumination (X) doubled, trebled, etc.

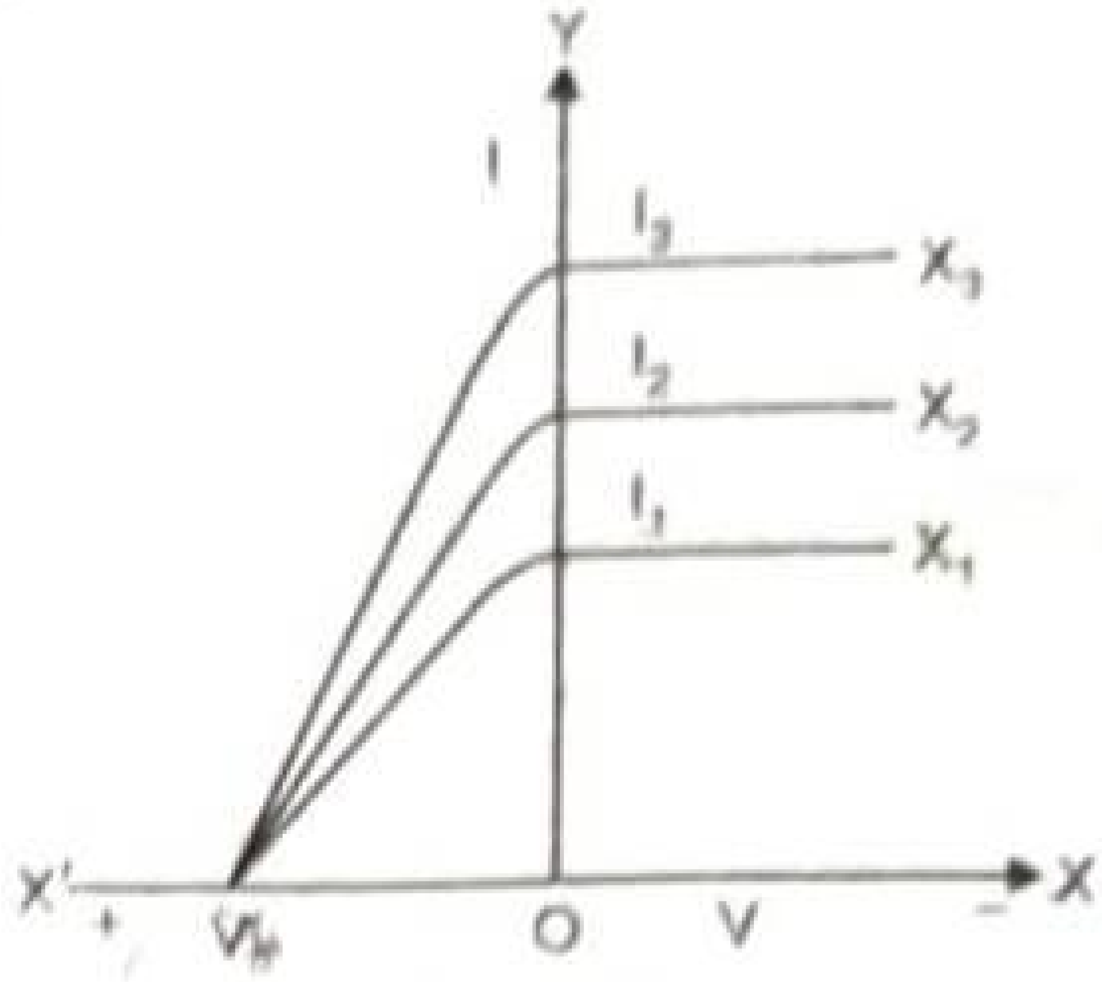


Fig 6.5

Graph. Fig. 6.5 shows the relation between I and V.

- For a given intensity of illumination (X_1), there is no photoelectric current when the positive potential on the cathode is greater than a critical value $+V_0$. At potentials just less than $+V_0$, a small current is produced. As the potential decreases to zero, the current rapidly increases and reaches a maximum value when $V = 0$. No further increases in current are observed when V becomes negative. Hence I_1 is the maximum current due to illumination of intensity X_1.
- When the intensity is doubled (X_2) as in (2), the maximum current I_2 is double I_1: But the critical potential V_0 is the same as before.
- When the intensity of illumination is increased three fold (X_3), the corresponding maximum current increases proportionately, but the critical potential remains unchanged.

Result. The maximum current I_m is proportional to the intensity of illumination X, *i.e.*, $I_m \propto X$. The critical potential V_0 is independent of the intensity.

(ii) **Relation between velocity of photoelectrons and the frequency of light.** Several monochromatic radiations of wavelengths λ_1, λ_2, λ_3, etc., are allowed to fall on the emitter. The intensity of illumination for each wavelength is adjusted to give the same value of I_m in each case. In each case, the photoelectric current I for different values of V is determined.

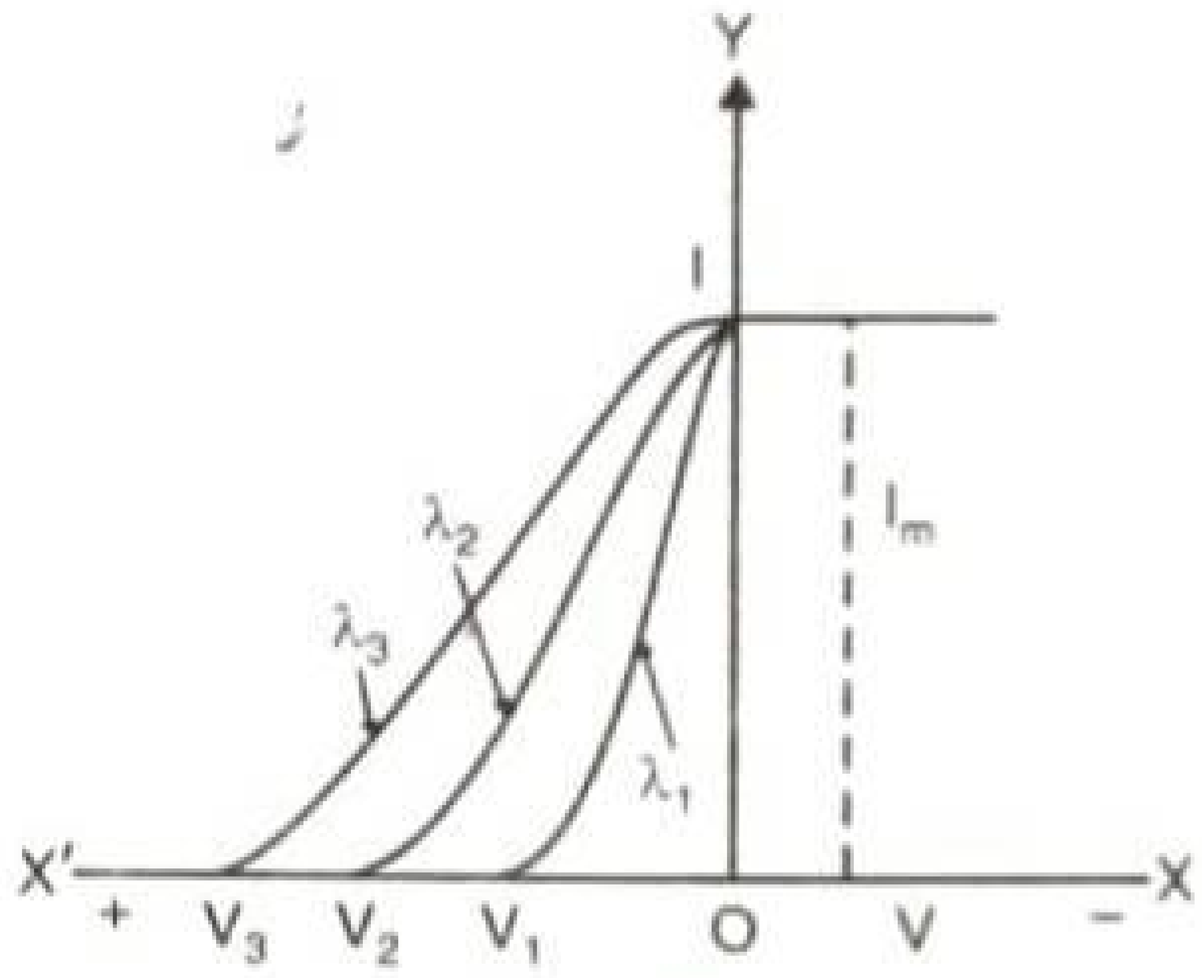


Fig. 6.6

Graph. Fig. 6.6 shows the nature of the curves.

- If $\lambda_1 > \lambda_2 > \lambda_3$, the critical potentials are $V_1 < V_2 < V_3$.
  Hence, as the wavelength of light increases, the critical retarding potential decreases. This means that the maximum K.E. of the photoelectrons (given by $\frac{1}{2} mv_m^2 = eV_0$) increases with increasing frequency of light.
- Since V_0 is independent of the intensity of illumination, we conclude that the velocity and K.E. of photoelectrons are independent of the intensity of illumination, but dependent on the frequency of the incident light.

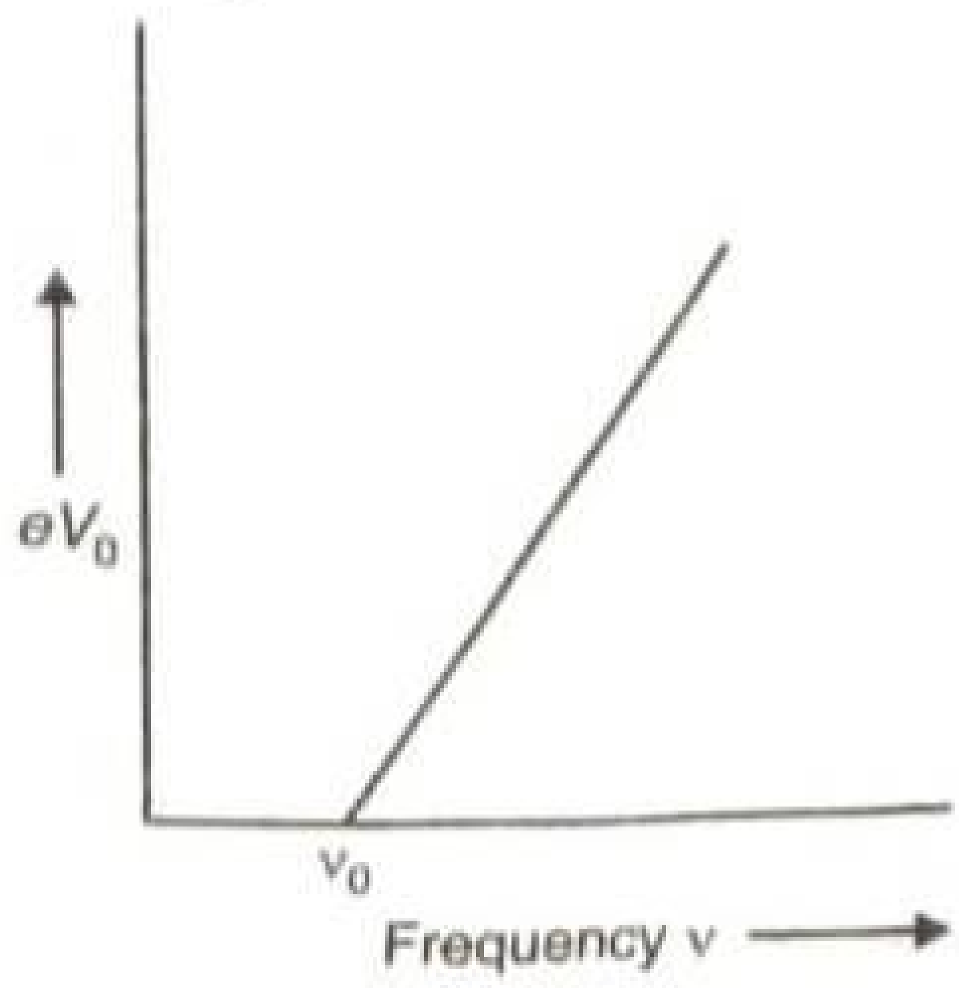


Fig. 6.7

A linear relation is found to exist between the maximum energy of emission (eV_0) and the frequency (ν) of the light. If eV_0 is plotted against ν, we get, for any emitting surface a straight line (Fig. 6.7), whose intercept on the frequency axis gives the threshold frequency $\nu_0 = c/\lambda_0$ for this surface. The slope of the straight line is

Variation of photoelectric current with cathode potential

Lenard first studied the relation between current and the potential applied to C. When the cathode potential was several volts positive, the current was zero. When V was +2 volts, there was a feeble current showing that a few particles possessed enough velocity to overcome the retarding potential of 2 volts. When the potential was further decreased, the current increased and reached a saturation value for −20 volts. Fig. 6.3 shows the variation of photoelectric current with cathode potential.

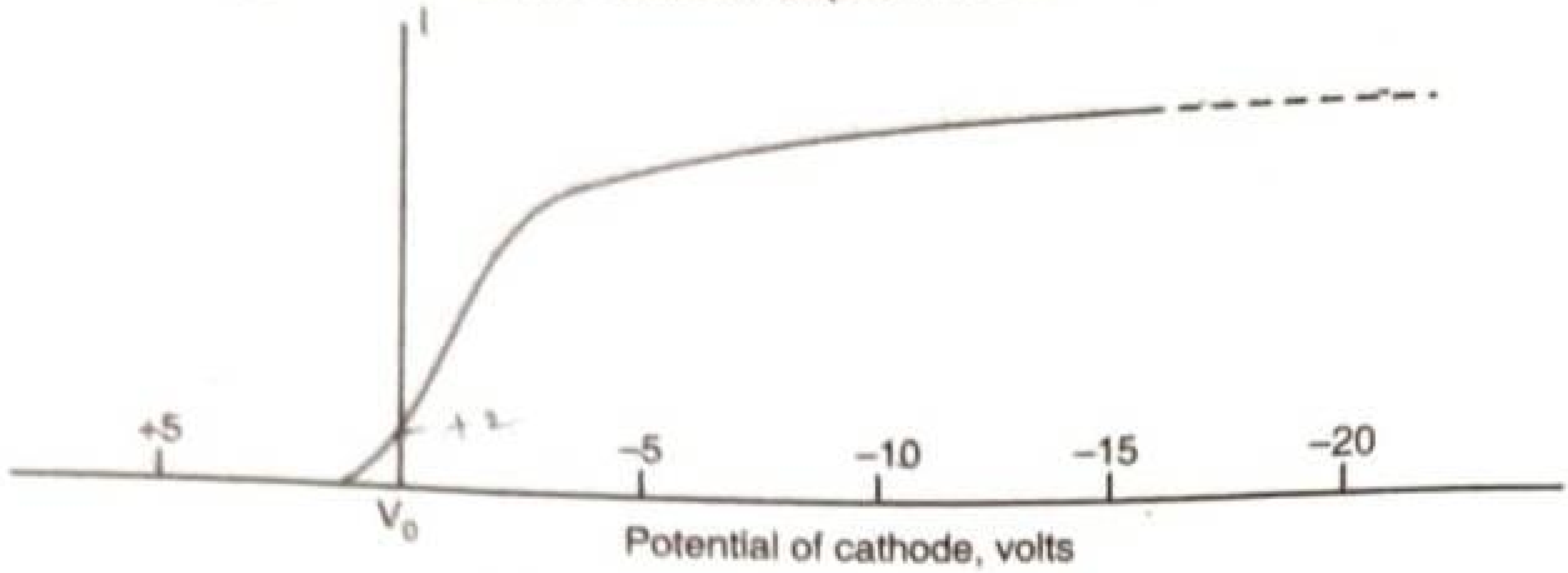


Fig. 6.3

Determination of e/m. After this preliminary investigation, Lenard applied to C a negative potential V, very large compared to the potential of 2 volts. The velocity imparted by the accelerating potential is so large that the velocity of the particles in the act of emission is negligible in comparison to it. Let V be the accelerating potential and v the velocity acquired by the photoelectrons. Then,

$$\frac{1}{2}mv^2 = eV \qquad ...(1)$$

where e is the charge and m the mass of the photoelectron.

Let R be the radius of the circular path described by the photoelectrons in the region of uniform magnetic field of strength B.

Then $$\frac{mv^2}{R} = Bev.$$

$\therefore$ $$v = \frac{BeR}{m} \qquad ...(2)$$

Substituting this value of v in Eq. (1), $\frac{1}{2}m\,(BeR/m)^2 = eV$

$\therefore$ $$\frac{e}{m} = \frac{2V}{B^2R^2} \qquad ...(3)$$

Knowing V, B and R, e/m is calculated. Lenard found the value of e/m to be the same as that for electrons. *This clearly shows that the photoparticles are nothing but electrons.*

6.3 RICHARDSON AND COMPTON EXPERIMENT

Apparatus: The apparatus used by them is shown in Fig. 6.4. The emitter of photoelectrons (C) is a small strip of the metal under study and is kept at the centre of a spherical glass bulb B. B is silvered on the inner side and can be evacuated through the tube T. The silver coating on the inside of the bulb serves as the anode A and is connected to an electrometer E which measures the photoelectric current. Monochromatic light L is made to pass through a quartz window W and fall on C. C can be maintained at any desired potential V relative to A and this potential can be read with a voltmeter.

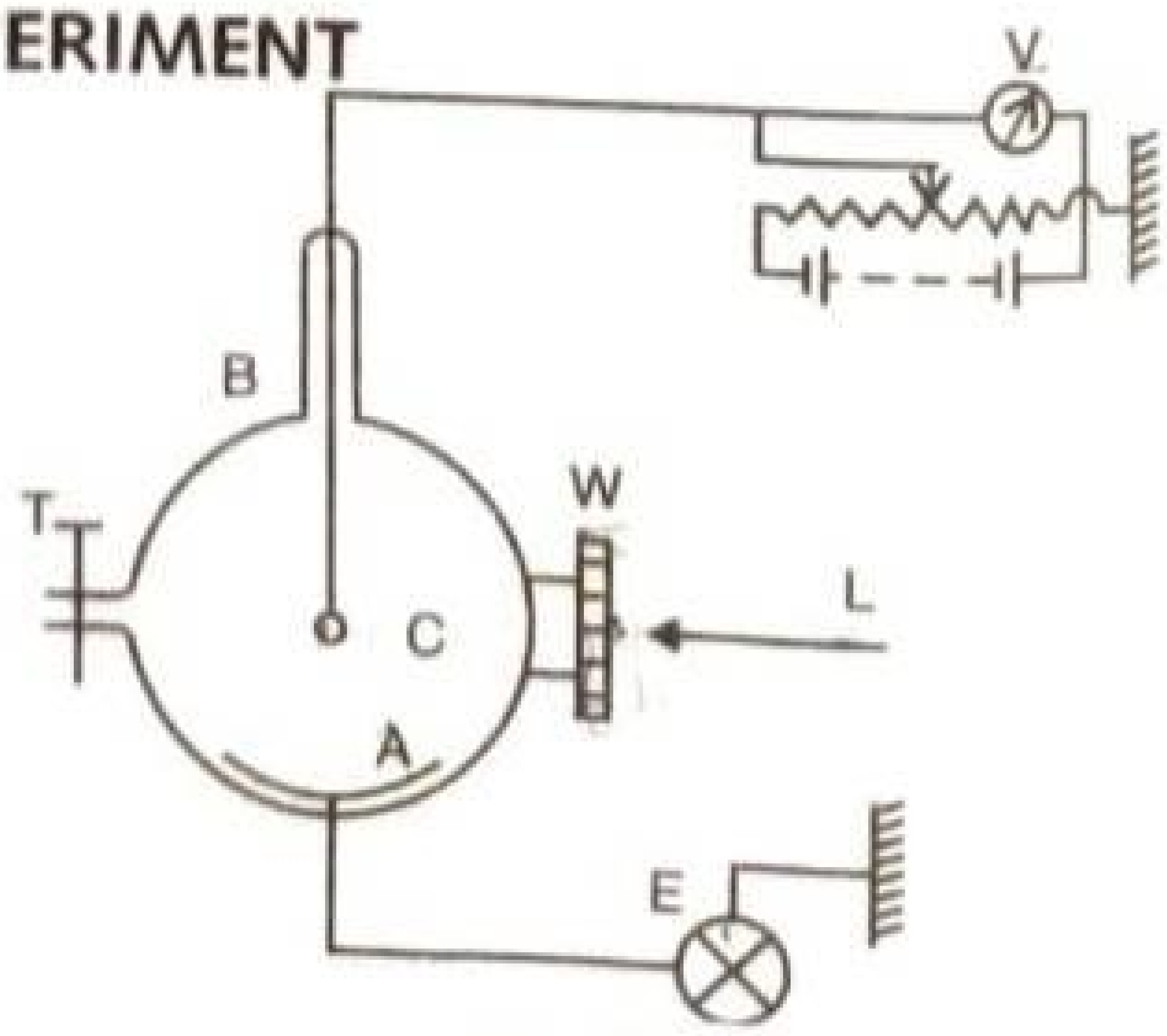


Fig. 6.4

This is *Wien's law*.

- Planck's formula reduces to Rayleigh Jean's formula for longer wavelengths.

When λ is large, $e^{hc/\lambda kT} \approx 1 + (hc/\lambda kT)$.

Hence Planck's law reduces to

$$E_\lambda \, d\lambda = 8\pi \frac{hc\lambda^{-5}}{(hc/\lambda kT)} d\lambda = 8\pi kT\lambda^{-4} \, d\lambda$$

This is *Rayleigh-Jeans formula*.

EXERCISE

1. The velocity of photoelectron depends upon the .... of the incident photon only. (*B.U., April 2012*) [**Ans.** Freq
2. .... is known as the Einstein's photoelectric equation. (*B.U., April 2012*) [**Ans.** $h\nu = \phi +$ 1.
3. At threshold frequency, the kinetic energy of emitted photoelectron is .... (*B.U., April 2014*) [**Ans**
4. The photoelectric work function ϕ = ... (*B.U., April 2013*) [**Ans.** ϕ
5. Millikan's experiment is based on ....
   (*a*) developing potential (*b*) high potential (*c*) stopping potential (*d*) negative potent
   (*B.U., April 2013*) [**A**
6. In photoconductive cell, as the intensity of radiation increases, ... of semiconductor decreases.
   (*a*) elasticity (*b*) hardness (*c*) resistance (*d*) none of these
   (*B.U., April 2014*) [**A**
7. Explain the Richardson and Compton experiment to study the photoelectric phenomena. (*B.U., April*
8. State and explain laws of photoelectric emission. (*B.U., April*
9. Derive Einstein's photoelectric equation. (*B.U., April*
10. Calculate the work function of sodium, in electron-volts, given that the threshold wavelen 6800 Å, and $h = 6.625 \times 10^{-34}$ Js. (*B.U., April*

    [**Sol.** $\phi = h\nu_0 = hc/\lambda_0$

    Here, $h = 6.625 \times 10^{-34}$ Js; $c = 3 \times 10^8$ ms^{-1} and $\lambda_0 = 6800 \times 10^{-10}$ m.

    $$\therefore \quad \phi = \frac{(6.625\times10^{-34})(3\times10^8)}{(6800\times10^{-10})} \text{ J} = \frac{(6.625\times10^{-34})(3\times10^8)}{(6800\times10^{-10})(1.6\times10^{-19})} \text{ eV}$$

    $= 1.827$ eV.]
11. The photoelectric threshold for a metal is 3000 Å. Find the kinetic energy of an electron ejecte it by radiation of wavelength 1200 Å.

    [**Sol.** K.E. of the electron $= \frac{1}{2}mv^2 = h(\nu - \nu_0)$

    $$= \frac{hc(\lambda_0 - \lambda)}{\lambda\,\lambda_0} = \frac{(6.62\times10^{-34})(3\times10^8)(1800\times10^{-10})}{(3000\times10^{-10})(1200\times10^{-10})} = 9.93 \times 10^{-19} \text{ J} = 6.2 \text{ eV}]$$
12. Discuss the relation between photoelectric current and retarding potential in photoelectric effect (*B.U., April*
13. With necessary theory, discuss the Millikan's experimental verification of Einstein's photoelectric tion. (*B.U., April*
14. Explain the action of a photoelectric cell. (*B.U., April*
15. Describe construction and working of photovoltaic cell. (*B.U., April*
16. What is photoconductive cell? Explain its working and important applications. (*B.U., April*
17. Discuss the distribution of energy in the spectrum of a black body and its results. (*B.U., April*
18. Derive an expression for Planck's law of radiation. (*B.U., April*

6.1 INTRODUCTION

Whenever light or electromagnetic radiations (such as X-rays, Ultraviolet rays) fall on a metal surface, it emits electrons. *This process of emission of electrons from a metal plate, when illuminated by light of suitable wavelength, is called the photoelectric effect.* The electrons emitted are called the *photoelectrons*. In the case of alkali metals, photoelectric emission occurs even under the action of visible light. Zinc, cadmium etc., are sensitive to only ultraviolet light.

The Nature of Photo-particles

Experimental arrangement. The apparatus consists of two plates *A* and *C* placed in an evacuated quartz bulb (Fig. 6.1).

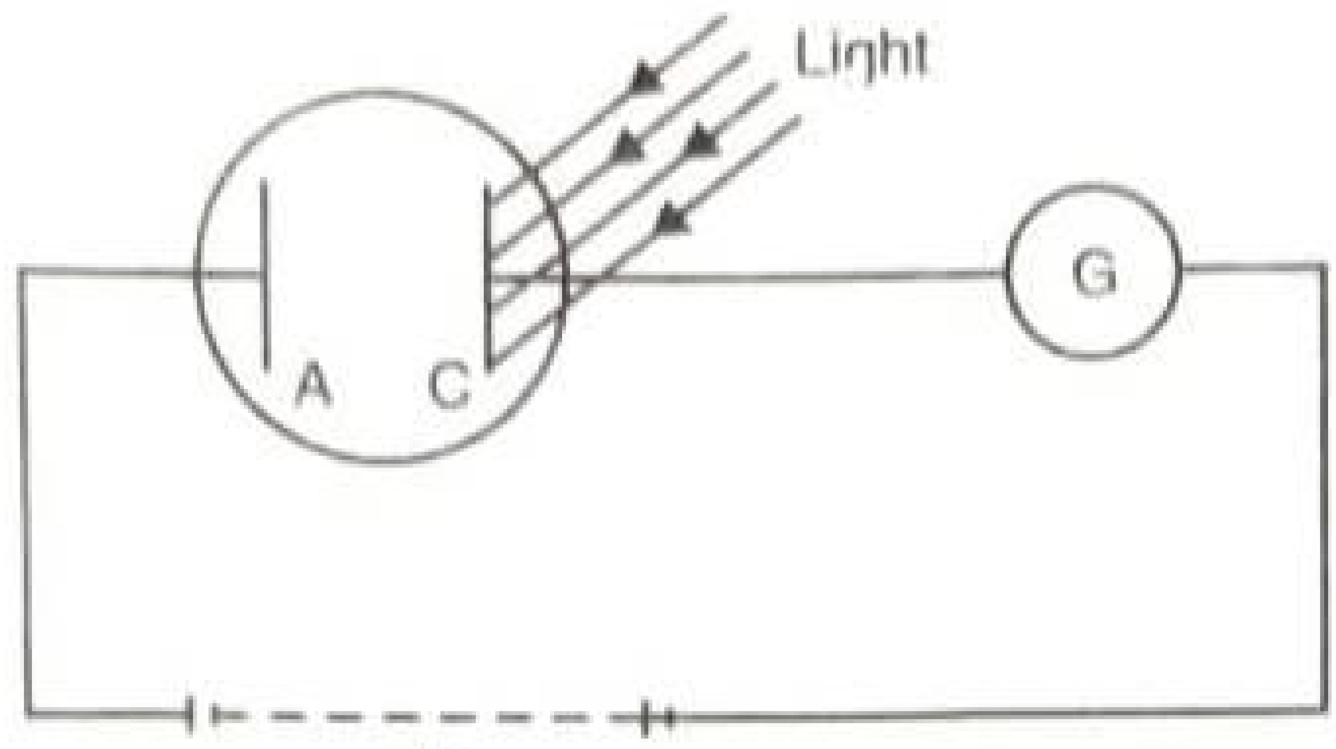


Fig. 6.1

The galvanometer (*G*) and battery (*E*) are connected as shown. When ultraviolet light is incident on the negative plate *C*, a current flows in the circuit as indicated by the galvanometer. But when light falls on the positive plate *A*, there is no current in the circuit. These observations show that photo-particle must be negatively charged.

6.2 LENARD'S METHOD TO DETERMINE E/M FOR PHOTOELECTRONS

Apparatus. The apparatus used is shown in Fig. 6.2. It consists of a glass tube *G* which can be evacuated through the side tube *T*. Ultraviolet light passes through a quartz window *W* and falls on an aluminium cathode *C* enclosed in *G*. An earthed metal screen *A* with a small central hole forms the anode. The cathode (*C*) can be maintained at a desired potential, positive or negative relative to the anode *A*. P_1 and P_2 are small metal electrodes connected to electrometers E_1 and E_2.

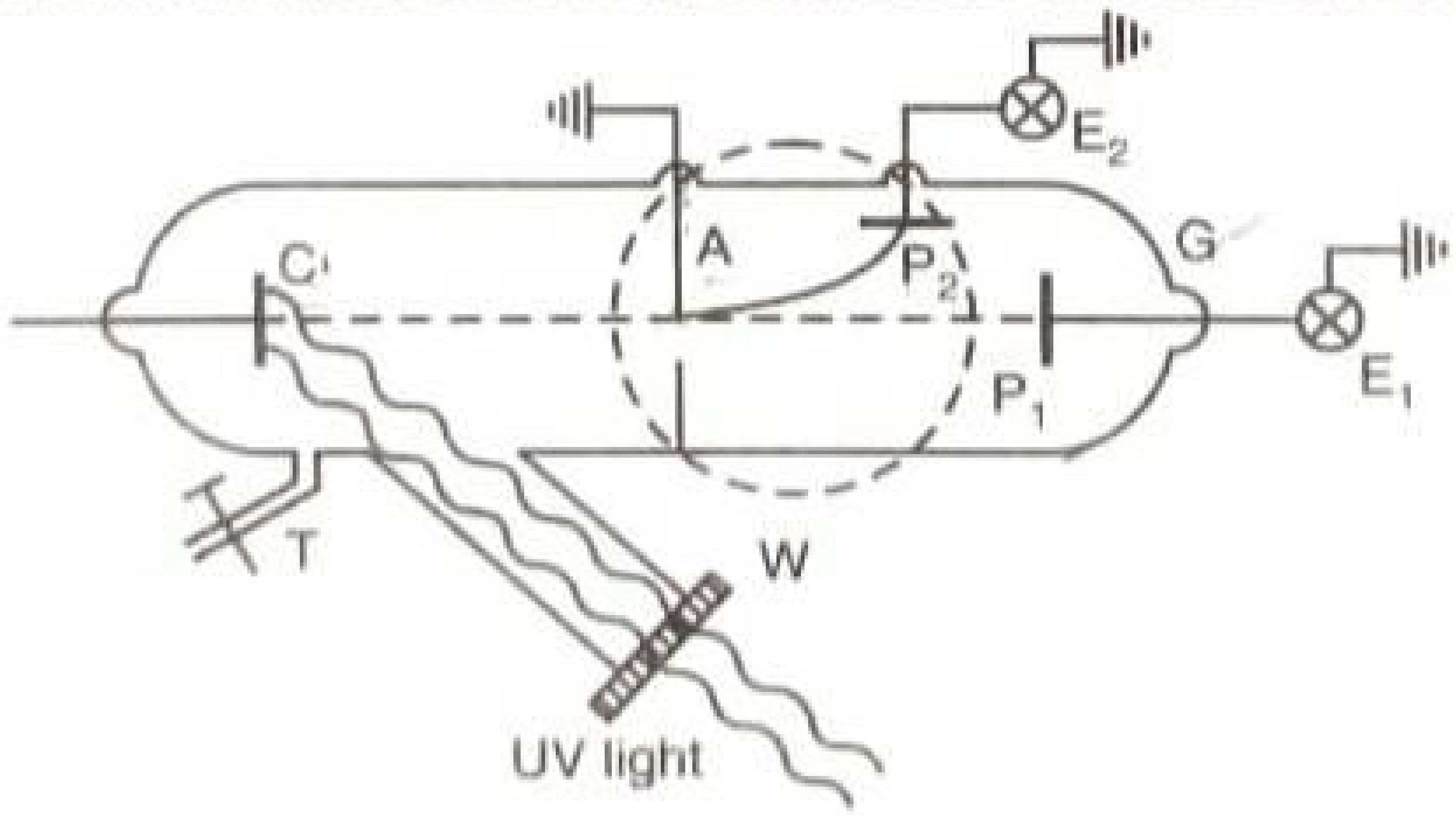


Fig. 6.2

Working. When *C* is raised to a negative potential and illuminated, negatively charged particles are produced and accelerated towards the anode. A few particles pass through the hole in *A* and proceed with uniform velocity to P_1. Their arrival at P_1 is indicated by E_1. By applying a uniform magnetic field *B* (represented by the dotted circle) perpendicular to the plane of the figure and directed towards the reader, the photoelectrons can be deflected towards P_2. Their arrival at P_2 is indicated by the deflection they produce in E_2.

Putting $\varepsilon/kT = x$,

$$N = N_0 + N_0 e^{-x} + N_0 e^{-2x} + \ldots\ldots\ldots\ldots N_0 e^{-rx}$$

$$N = \frac{N_0}{1-e^{-x}} \qquad \ldots(1)$$

The total energy of Planck's resonators is

$$E = 0 \times N_0 + \varepsilon \times N_0 e^{-x} + 2\varepsilon \times N_0 e^{-2x} + \cdots + r\varepsilon \times N_0 e^{-rx} + \cdots$$

$$Ee^{-x} = \varepsilon N_0 e^{-2x} + 2\varepsilon N_0 e^{-3x} + \cdots + r\varepsilon N_0 e^{-(r+1)x}$$

Subtracting, $E(1-e^{-x}) = \varepsilon N_0 e^{-x} + \varepsilon N_0 e^{-2x} + \varepsilon N_0 e^{-3x} + \cdots$

$$= \frac{\varepsilon N_0 e^{-x}}{1-e^{-x}}$$

$$\therefore \quad E = \frac{\varepsilon N_0 e^{-x}}{(1-e^{-x})^2} \qquad \ldots(2)$$

$$\left.\begin{array}{l}\text{Average energy}\\ \text{of a resonator}\end{array}\right\} = \bar{\varepsilon} = \frac{E}{N} = \frac{\varepsilon e^{-x}}{1-e^{-x}} = \frac{\varepsilon}{e^{x}-1}$$

According to Planck's hypothesis, $\varepsilon = h\nu$. Further $\nu = c/\lambda$.

Hence,

$$\varepsilon = \frac{hc}{\lambda} \quad \text{and} \quad x = \frac{\varepsilon}{kT} = \frac{hc}{\lambda kT}$$

$$\therefore \quad \bar{\varepsilon} = \frac{hc/\lambda}{\left(e^{hc/\lambda kT}-1\right)} \qquad \ldots(3)$$

Number of oscillators per unit volume in the wavelength range λ and $\lambda + d\lambda = 8\pi\,\lambda^{-4}\,d\lambda$. ...(4)

Hence, energy density of radiation between wavelengths λ and $\lambda + d\lambda$ = (average energy of a Planck's oscillator) × (number of oscillators per unit volume).

$$E_\lambda\, d\lambda = \frac{hc/\lambda}{\left(e^{hc/\lambda kT}-1\right)} \times 8\pi\lambda^{-4}\, d\lambda$$

$$E_\lambda\, d\lambda = \frac{8\pi hc\lambda^{-5}}{\left(e^{hc/\lambda kT}-1\right)}\, d\lambda \qquad \ldots(5)$$

or

$$E_\nu d\nu = \frac{8\pi h\nu^3}{c^3\left(e^{h\nu/kT}-1\right)}\, d\nu \qquad \ldots(6)$$

Here $E_\nu\, d\nu$ is the energy density belonging to the range $d\nu$. Eq. (5) represents Planck's radiation law in terms of wavelength. Planck's formula fits the experimental curve very closely [Fig. 6.18].

- Planck's formula reduces to Wien's formula for small wavelengths. When λ is small, $e^{hc/\lambda kT}$ is large when compared to 1. Hence Eq. (5) reduces to

$$E_\lambda d\lambda = 8\pi\, h\, c\, \lambda^{-5}\, e^{-hc/\lambda kT}\, d\lambda \qquad \ldots(7)$$

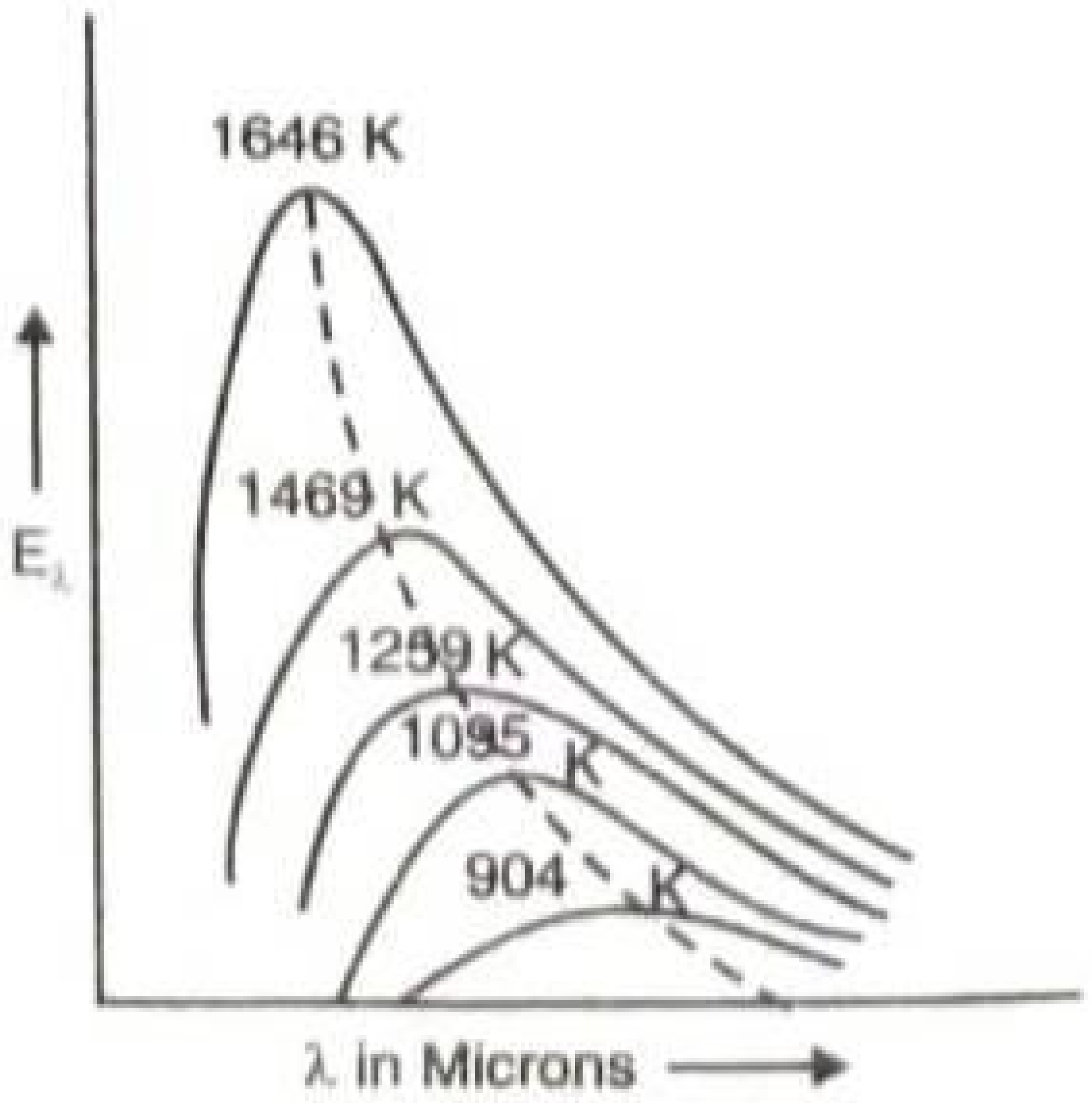


Fig. 6.18

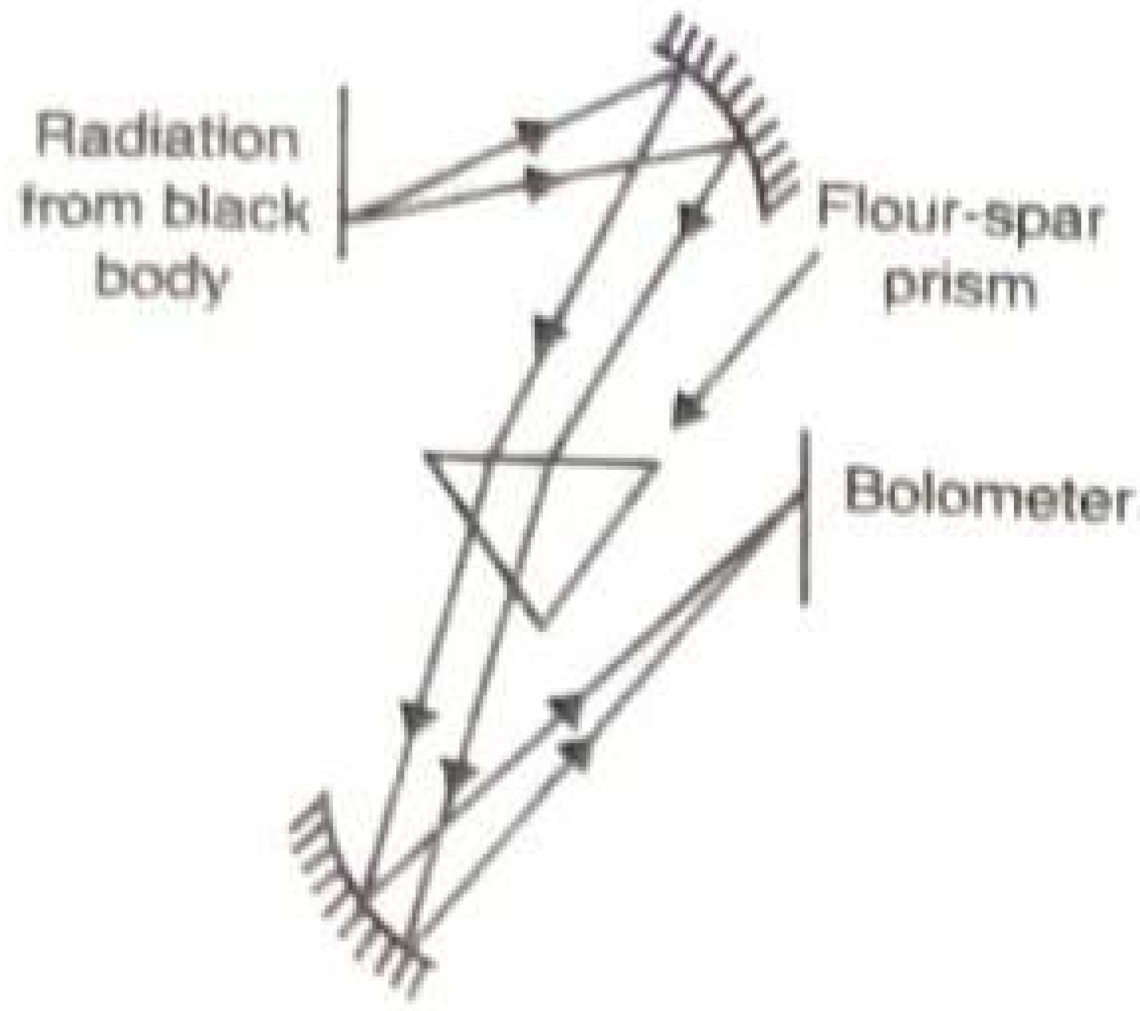


Fig. 6.16

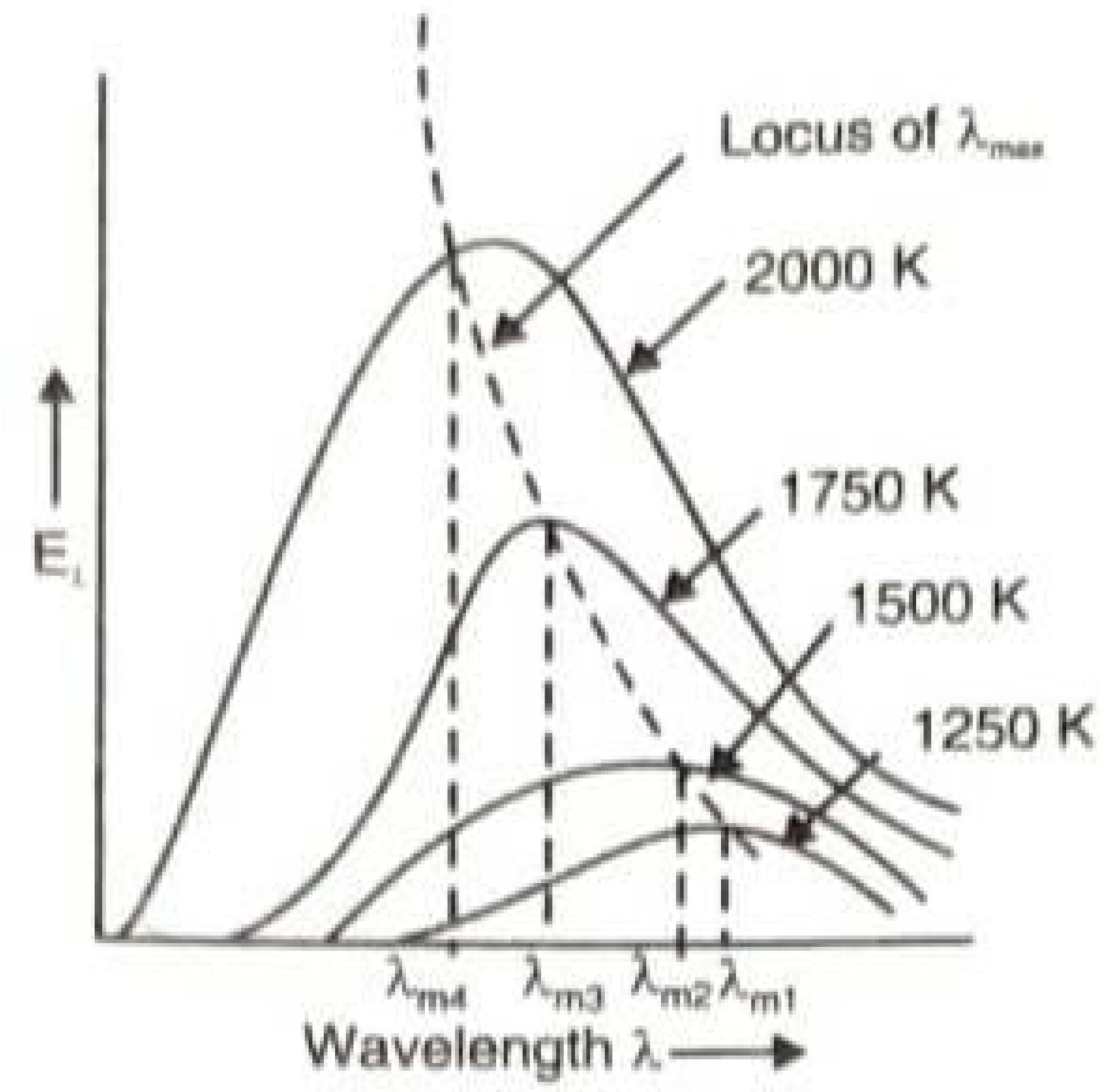


Fig. 6.17

(*iv*) Total energy emitted per unit area of the source per second at a given temperature is $\int_0^\infty E_\lambda \, d\lambda$. It will be represented by the total area between the curve for that temperature and the λ-axis. This area is found to be proportional to the fourth power of the absolute temperature. This verifies *Stefan's law*.

Wien's Displacement Law. *The wavelength of the most strongly emitted radiation in the continuous spectrum from a full radiator is inversely proportional to the absolute temperature of that body, i.e.,* $\lambda_m T = b$.

Here, b is Wien's constant 2.898×10^{-3} mK.

Planck's hypothesis. According to the classical theory of radiation, energy changes of radiators take place continuously. The classical theory failed to explain the experimentally observed distribution of energy in the spectrum of a black body. Planck succeeded in deriving a formula which agrees extremely well with experimental results. He discarded both the idea of radiation being a continuous stream as well as the law of equipartition of energy. He suggested the *quantum theory of radiation*. His assumptions are:

Max Planck (1858–1947)

(1) A black-body radiation chamber is filled up not only with radiation, but also with *simple harmonic oscillators* or *resonators* of molecular dimensions. They can vibrate with all possible frequencies.

(2) The oscillators or resonators cannot radiate or absorb energy continuously. But an oscillator of frequency ν can only radiate or absorb energy in units or quanta of magnitute $h\nu$. h is a universal constant called Planck's constant. The emission of radiation corresponds to a decrease and absorption to an increase in the energy and amplitude of an oscillator.

Derivation of Planck's law of radiation. Let N be the total number of Planck's resonators and E their total energy. Then, average energy per oscillator $= \bar{\varepsilon} = E / N$.

$$N = N_0 + N_0 e^{-\varepsilon/kT} + N_0 e^{-2\varepsilon/kT} + \cdots + N_0 e^{-r\varepsilon/kT} + \cdots$$

Here

$$N_0 = \text{number of resonators having 0 energy.}$$
$$N_0 e^{-\varepsilon/kT} = \text{number of resonators having energy } \varepsilon,$$
$$N_0 e^{-2\varepsilon/kT} = \text{number of resonators having energy } 2\varepsilon,$$
$$N_0 e^{-r\varepsilon/kT} = \text{number of resonators having energy } r\varepsilon \text{ and so on.}$$

the ejection of one or more secondary electrons from the surface. Suppose that a photoelectron striking dynode 1 produces x electrons by secondary emission. These electrons are then directed towards dynode 2 by making its potential higher than that of dynode 1. Suppose x electrons are again ejected by secondary emission for each incident electron. Then, for each electron emitted by the photosensitive plate, there are now x^2 electrons and so on. If there are several dynodes, each at a potential higher than the preceding one, an avalanche of electrons reaches the collector plate A. A strong current then flows in the outer circuit. This device is used to amplify very weak light signals.

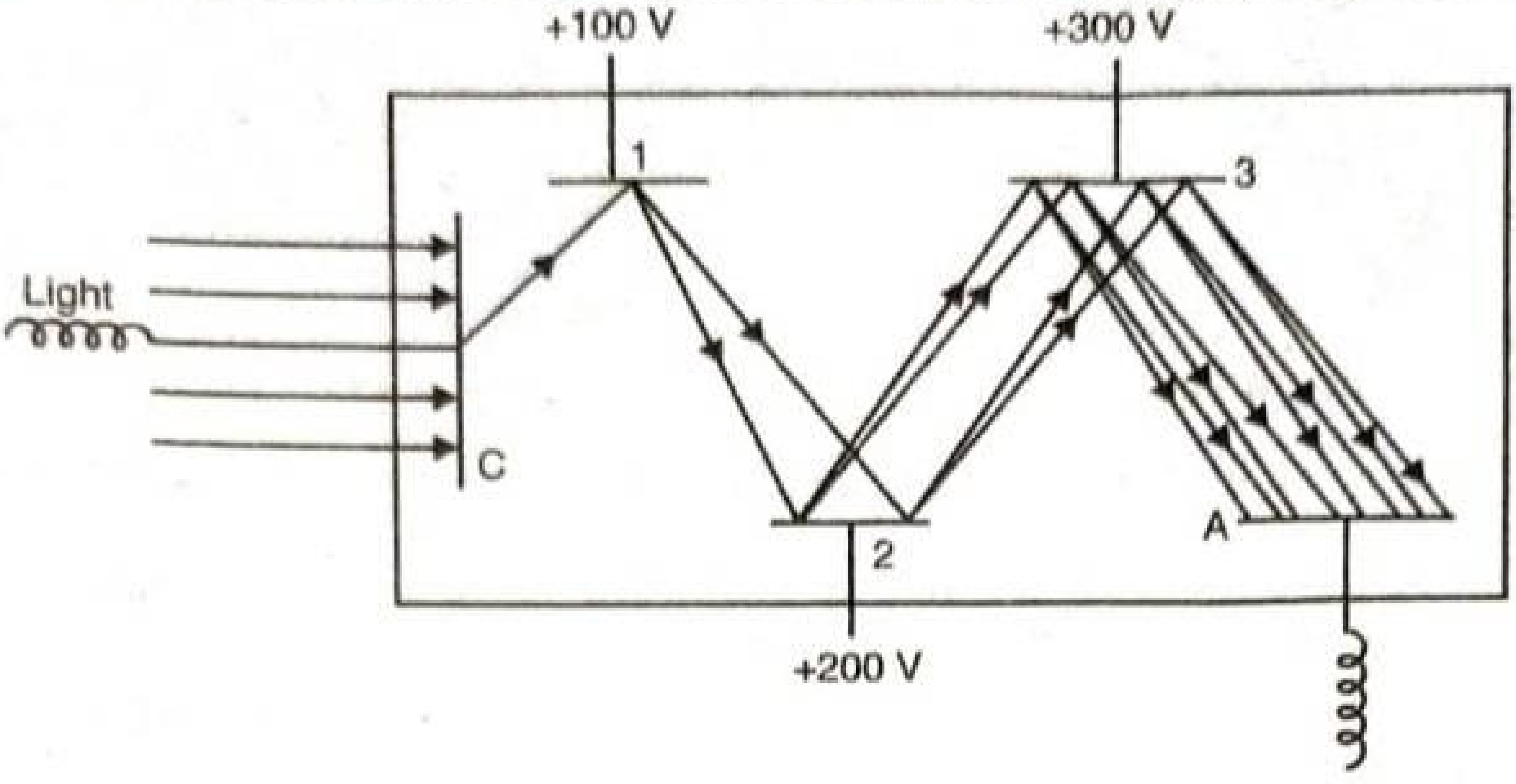


Fig. 6.15

(*iii*) Photoelectric cells are used to compare the illuminating powers of two light sources. They are also used in the measurement of the intensity of illumination of a light source.

(*iv*) **Sound reproduction in films.** The film is provided with a sound track at one edge. Light passing through the sound track of the film falls on a photocell. Current is produced, which fluctuates correspondingly with the intensity of sound recorded in the sound track. The current impulses are converted to sound by speakers.

(*v*) **Automatic operation of street lights.** A photoelectric cell, located in a street light circuit, switches off the street light when sunlight is incident on the cell. When sunlight fades and it becomes dark, the photoelectric cell switches on the street lights.

6.7 PLANCK'S QUANTUM THEORY

The distribution of energy in the spectrum of a black body. If the radiation emitted by a black body at a fixed temperature is analysed by means of a suitable spectroscopic arrangement, it is found to spread up into a continuous spectrum. The total energy is not distributed uniformly over the entire range of the spectrum.

Experimental arrangement. The distribution of energy in various parts of the spectrum was experimentally studied by Lummer and Pringsheim. The radiation from the black body was rendered into a parallel beam by the concave mirror [Fig. 6.16]. It is then allowed to fall on a prism of fluorspar to resolve it into a spectrum. The spectrum is brought to focus by another concave mirror on to a linear bolometer. The bolometer is connected to a galvanometer. The deflections in the galvanometer corresponding to different λ are noted by rotating the prism table. Then curves are plotted for E_λ versus λ. The experiment is done with the black body at different temperatures. The curves obtained are shown in Fig. 6.17.

Results. (*i*) At any given temperature, E_λ first increases with λ, reaching a maximum value corresponding to a particular wavelength λ_m and then decreases for longer wavelengths.

(*ii*) The value of E_λ for any λ increases as temperature increases.

(*iii*) The wavelength corresponding to the maximum energy shifts to shorter wavelength side as the temperature increases. This confirms Wien's displacement law $\lambda_m T =$ constant.

proportional to the intensity of light. No external battery is required to operate a photovoltaic cell as the cell itself generates an e.m.f.

(iii) Photoconductive Cell.

Principle: The photoconductive cell is based on the principle that the electrical resistance of a semiconductor material decreases with the increase of intensity of radiation incident upon it and conductivity is increased.

The photoconductive materials used are cadmium sulfide (CdS) and cadmium selenide (CdSe).

Construction: Fig. 6.13 shows the construction and graphical symbol of a photoconductive cell. It is a two-terminal semiconductor device. The terminal resistance varies (linearly) with the intensity of the incident light. A thin layer of the photoconductive material connected between terminals is simply exposed to the incident light energy.

Working: As the illumination on the device increases in intensity, the energy state of a larger number of electrons in the structure will also increase because of the increased availability of the photon packages of energy. The result is an increasing number of relatively "free" electrons in the structure and a decrease in the terminal resistance.

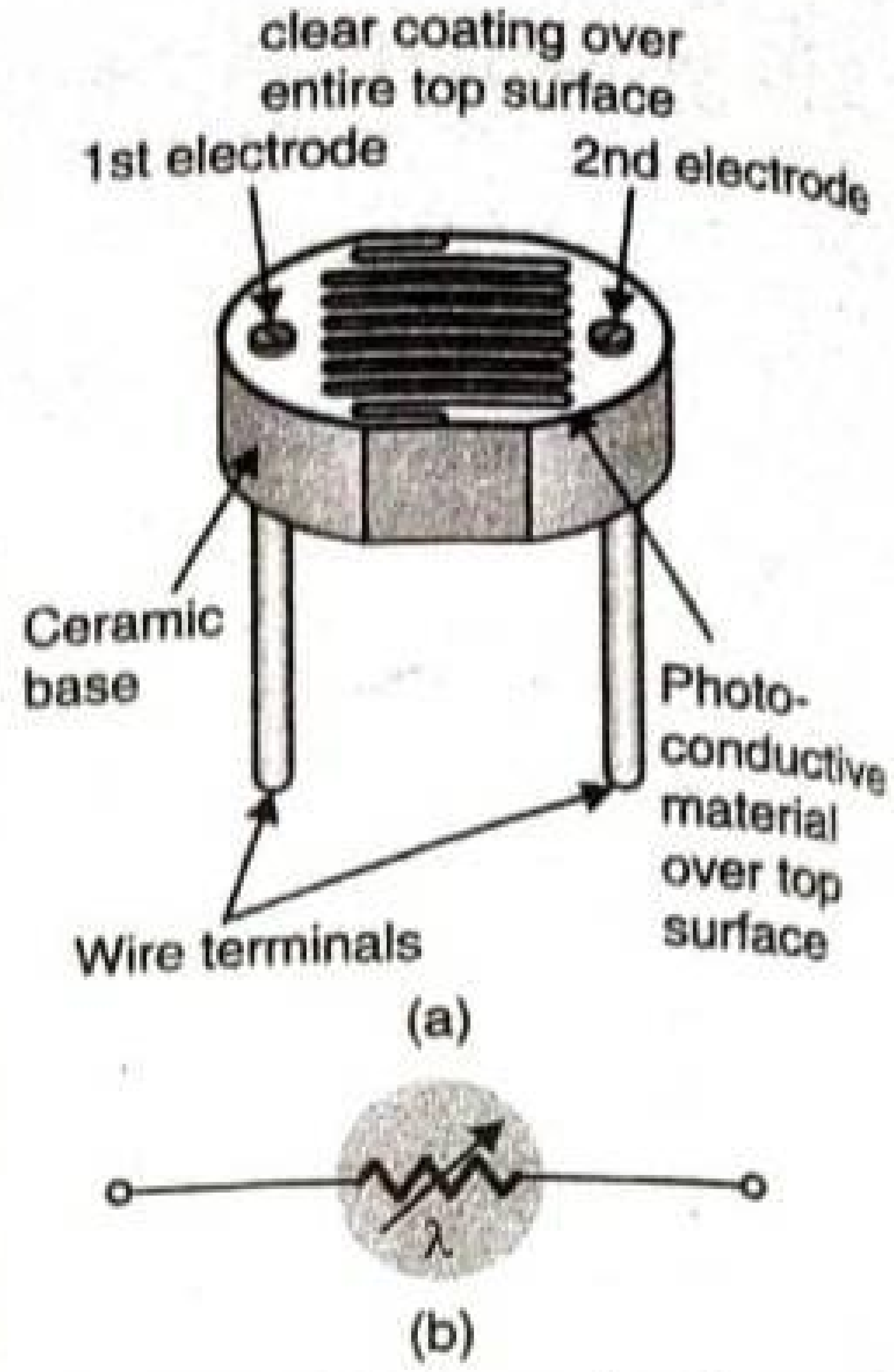


Fig. 6.13. Photoconductive cell: (a) construction; (b) symbol.

Fig. 6.14 shows the *terminal characteristics* of a photoconductive cell.

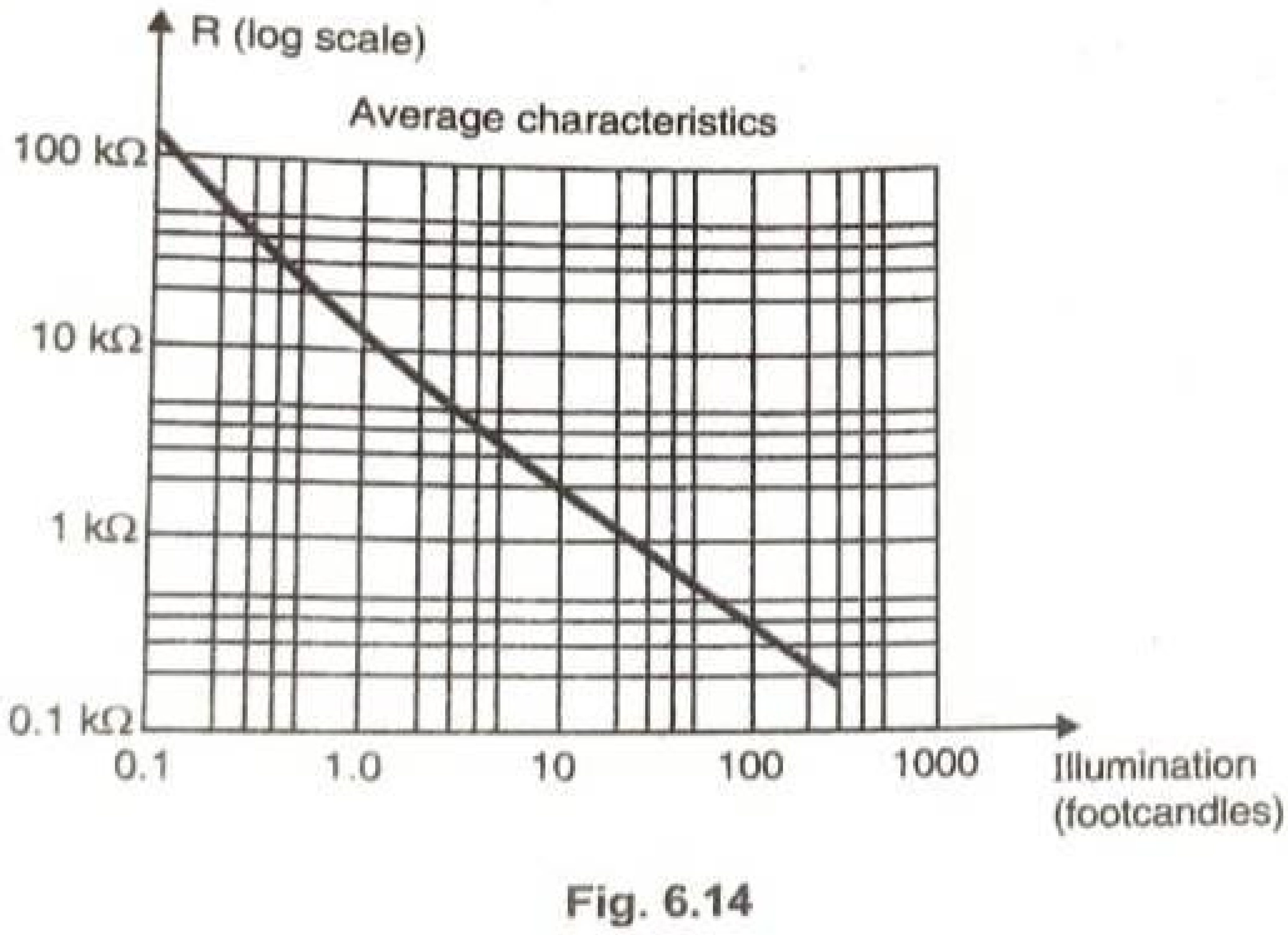


Fig. 6.14

Note the linearity (when plotted using a log–log scale) of the resulting curve and the large change in resistance (100 kΩ → 100 Ω) for the indicated change in illumination.

Applications of Photoelectric cells. (i) *Exposure meters in photography.* An exposure meter is a device to calculate the correct time of exposure. The photoelectric cell in the instrument produces a current proportional to the light falling on it. The current operates a galvanometer, the scale of which is calibrated to read the time of exposure.

(ii) Photo-multiplier. It is based on the principle of *secondary emission.* When light strikes the surface of photosensitive metal plate C, it causes the ejection of photoelectrons from it (Fig. 6.15). These electrons are then attracted to a metal surface called a dynode, by setting a P.D. between the cathode C and the dynode 1. High energy electrons striking a metal surface can cause

This is known as *Compton wavelength.*

Case 3. When $\theta = 180°$, $\cos\theta = -1$ and hence $d\lambda = 2h/m_0c = 0.0485$ Å. $d\lambda$ has the maximum value at $\theta = 180°$.

Experimental verification. Monochromatic X-rays of wavelength λ are allowed to fall on a scattering material like a small block of carbon (Fig. 5.19). The scattered X-rays are received by a Bragg spectrometer and their wavelength is determined. The spectrometer can freely swing in an arc about the scatterer. The wavelength of the scattered X-rays is measured for different values of the scattering angle. The experimental results obtained by Compton are shown in Fig. 5.20. In the scattered radiation in addition to the incident wavelength (λ), there exists a line of longer wavelength (λ'). The "*Compton shift*" $d\lambda$ is found to vary with the angle at which the scattered rays are observed.

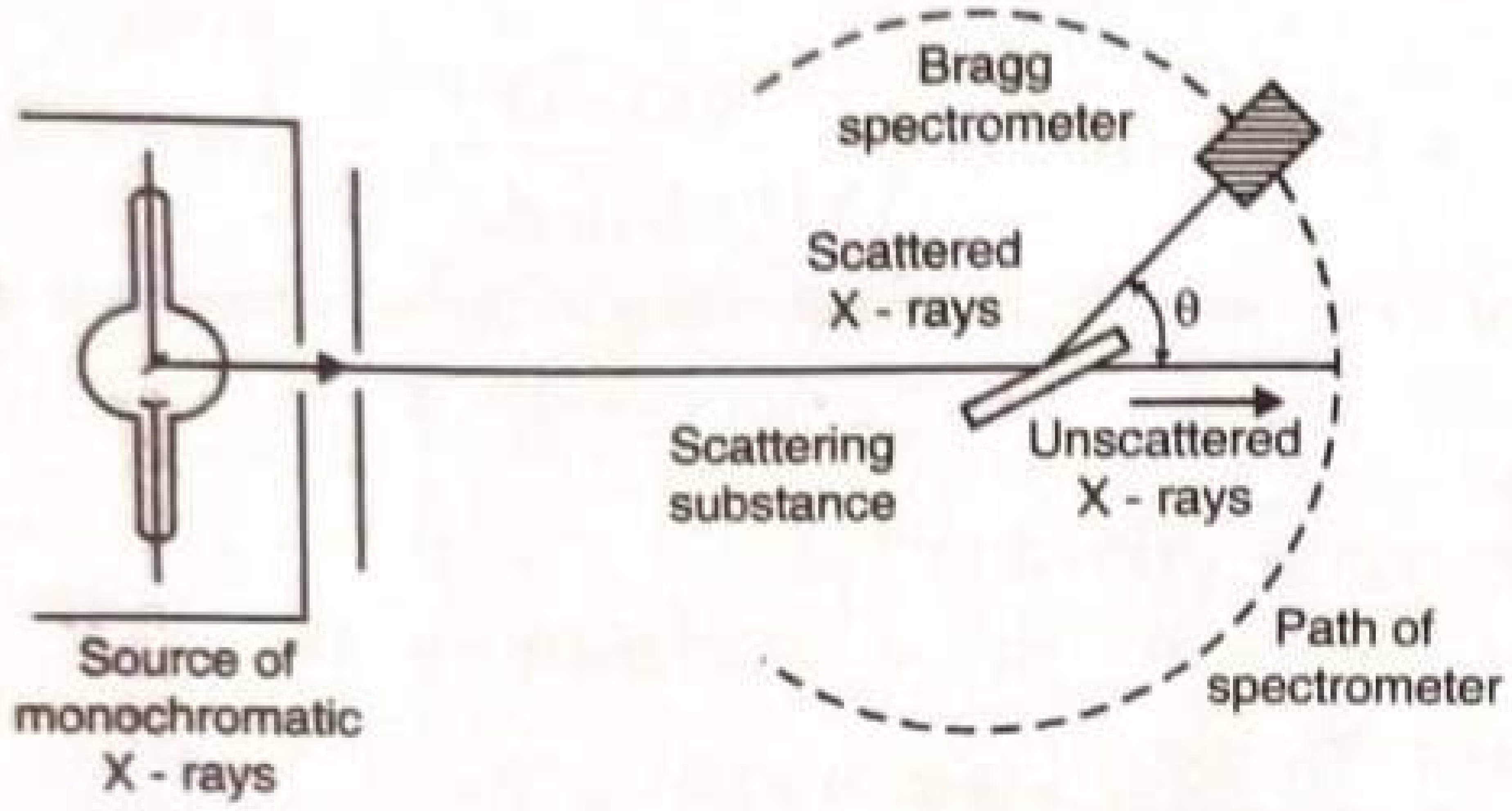


Fig. 5.19

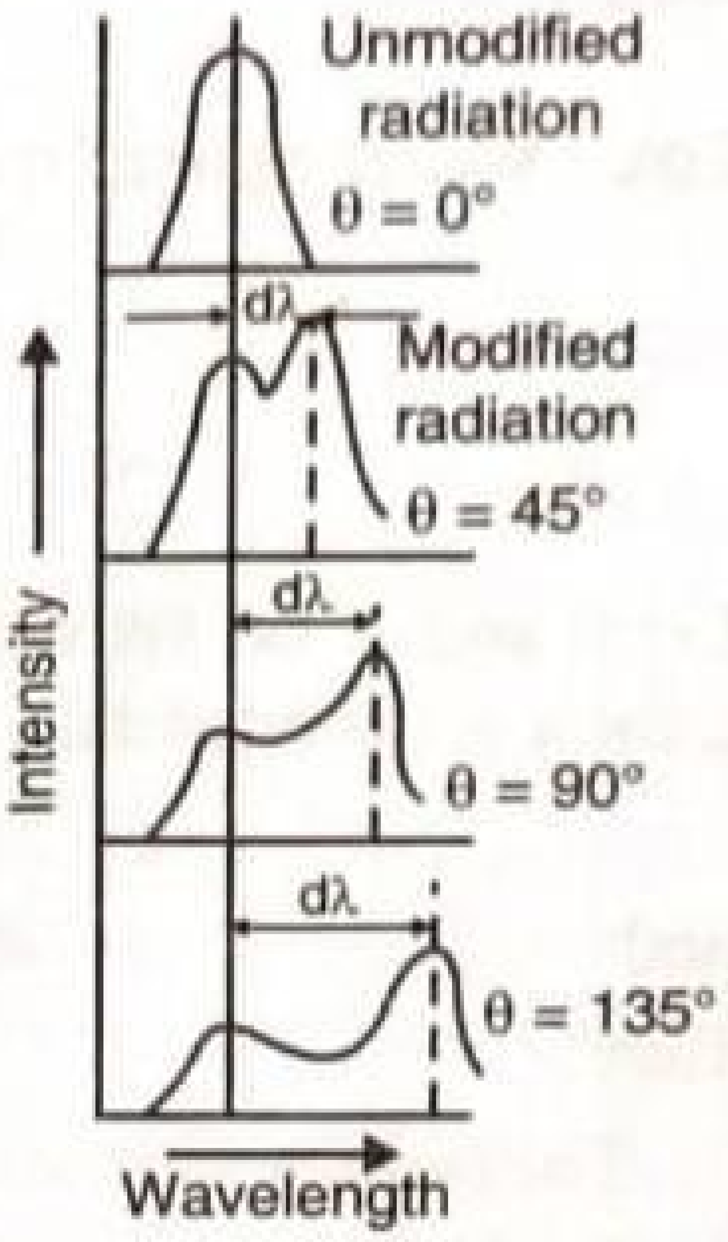


Fig. 5.20

Direction of Recoil electron. Dividing Eq. (5) by Eq. (4), we get

$$\tan\phi = \frac{h\nu'\sin\theta}{h(\nu-\nu'\cos\theta)} = \frac{\nu'\sin\theta}{(\nu-\nu'\cos\theta)}. \qquad ...(12)$$

Using Eq. (10), we get

$$\frac{1}{\nu'} = \frac{1}{\nu}+\frac{h}{m_0c^2}(1-\cos\theta) = \frac{1}{\nu}+\frac{h}{m_0c^2}.\,2\sin^2\frac{\theta}{2}$$

or

$$\nu' = \frac{\nu}{1+\left(\frac{h\nu}{m_0c^2}\right)2\sin^2\frac{\theta}{2}} = \frac{\nu}{1+2\beta\sin^2\left(\frac{\theta}{2}\right)} \text{ where } \beta = \frac{h\nu}{m_0c^2} \qquad ...(13)$$

Substituting this value of ν' in Eq. (12), we get

$$\tan\phi = \frac{\nu\sin\theta\Big/\left[1+2\beta\sin^2\left(\frac{\theta}{2}\right)\right]}{\left[\nu-\left\{\nu\cos\theta\Big/\left(1+2\beta\sin^2\frac{\theta}{2}\right)\right\}\right]} = \frac{\cot\left(\frac{\theta}{2}\right)}{(1+\beta)}$$

$\therefore$

$$\tan\phi = \frac{\cot\left(\frac{\theta}{2}\right)}{1+\left(\frac{h\nu}{m_0c^2}\right)} \qquad ...(14)$$

Kinetic Energy of Recoil electron. The K.E. of recoil electron is the difference between the energies of incident and scattered photons, *i.e.*,

$$K.E. = h\nu - h\nu'.$$

$$K.E. = h\nu - h\left[\frac{\nu}{1+2\beta\sin^2(\theta/2)}\right] = h\nu\left[\frac{2\beta\sin^2(\theta/2)}{1+2\beta\sin^2(\theta/2)}\right] \quad \dots (15)$$

where $\beta = h\nu / m_0c^2$

EXAMPLE 1. *X-rays of wavelength 0.7080 Å are scattered from a carbon block through an angle of 90° and are analysed with a calcite crystal, the interplanar distance of whose reflecting planes is 3.13 Å. Determine the angular separation, in the first order, between the modified and the unmodified rays.*

SOL. Wavelength of the modified rays $= \lambda' = \lambda + \frac{h}{m_0 c}(1-\cos\theta)$

$$= 0.7080 \times 10^{-10}\,\text{m} + \left(\frac{6.63\times10^{-34}}{9.11\times10^{-31}\times3\times10^{8}}\right)\text{m} = 0.7323\,\text{Å}$$

Let θ and θ' be the angles of Bragg reflections corresponding to the wavelengths λ and λ'. Then, for $n = 1$ (first order),

$$2d\sin\theta = n\lambda = 0.7080\times10^{-10}\,\text{m}$$

and $$2d\sin\theta' = n\lambda' = 0.7323\times10^{-10}\,\text{m}.$$

Here, $$d = 3.13\times10^{-10}\,\text{m};\ \therefore\ \theta = 6°30' \text{ and } \theta' = 6°\,43'.$$

$\therefore$ The angular separation, in the first order, between the modified and unmodified rays $\Big\}$ $= \theta' - \theta = 13'$

EXERCISE

1. The minimum wavelength of continuous X-ray is given by .... (*B.U., April 2012*)

   [**Ans.** $\lambda_{min} = \frac{hc}{eV}$]

2. .... is the equation of Bragg's law. (*B.U., April 2012*) [**Ans.** $2d\sin\theta = n\lambda$]

3. According to Moseley's law, the frequency of a spectral line in X-ray spectrum is directly proportional to ....

   (*a*) Z (*b*) Z^2 (*c*) Z^3 (*d*) Z^4

   (*B.U., April 2014*) [**Ans.** (*b*)]

4. X-rays are more penetrative than visible light. Why? (*B.U., April 2011*)

   [**Ans.** Wavelength of X-rays is very small compared to the wavelength of visible light *i.e.*, energy of X-rays is very high compared to the energy of visible light. Hence X-rays are more penetrative than visible light.]

5. Explain the presence of unmodified line in Compton scattering. (*H.P.U. 1996*)

   [**Hint.** When a photon collides with a bound electron, its wavelength does not change.]

6. Explain why Compton effect is experimentally not observed for visible light rays. (*H.P.U. 2001*)

   [**Ans.** $\Delta\lambda = \frac{h}{m_0c}(1-\cos\theta) = 0.024\,(1-\cos\theta)$. $\Delta\lambda$ depends only on the scattering angle θ. Maximum value of $\cos\theta = -1$ when $\theta = 180°$. Hence the maximum value of $\Delta\lambda = 0.048$ Å. This means that Compton effect can be detected only for those radiations whose wavelength is not greater than a few Å. For visible light ($\lambda \approx 5000$ Å), $(\Delta\lambda)_{max}$ is only about 0.001% of the initial wavelength which cannot be detected.]

Considering the x and y components of the momentum and applying the principle of conservation of momentum,

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + mv\cos\phi \qquad ...(2)$$

and
$$0 = \frac{h\nu'}{c}\sin\theta - mv\sin\phi \qquad ...(3)$$

From (2), $mvc\cos\phi = h(\nu - \nu'\cos\theta)$ ...(4)

From (3), $mvc\sin\phi = h\nu'\sin\theta$ ...(5)

Squaring and adding (4) and (5),

$$m^2v^2c^2 = h^2(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2\cos^2\theta) + h^2\nu'^2\sin^2\theta$$
$$= h^2(\nu^2 - 2\nu\nu'\cos\theta) + h^2\nu'^2 = h^2(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2) \qquad ...(6)$$

From (1),
$$mc^2 = h(\nu - \nu') + m_0c^2$$

$\therefore$
$$m^2c^4 = h^2(\nu^2 - 2\nu\nu' + \nu'^2) + 2h(\nu - \nu')\,m_0c^2 + m_0^2c^4 \qquad ...(7)$$

Subtracting (6) from (7),

$$m^2c^2(c^2 - v^2) = -2h^2\nu\nu'(1-\cos\theta) + 2h(\nu - \nu')\,m_0c^2 + m_0^2c^4 \qquad ...(8)$$

The value of $m^2c^2(c^2 - v^2)$ can be obtained from the relativistic formula

$$m = \frac{m_0}{\sqrt{(1 - v^2/c^2)}}. \text{ Squaring,}$$

$$m^2 = \frac{m_0^2}{1 - v^2/c^2} = \frac{m_0^2c^2}{c^2 - v^2}$$

$\therefore$
$$m^2c^2(c^2 - v^2) = m_0^2c^4 \qquad ...(9)$$

From (8) and (9),

$$m_0^2c^4 = -2h^2\nu\nu'(1-\cos\theta) + 2h(\nu - \nu')\,m_0c^2 + m_0^2c^4$$

$\therefore$
$$2h(\nu - \nu')m_0c^2 = 2h^2\nu\nu'(1-\cos\theta)$$

or
$$\frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\theta) \text{ or } \frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\theta)$$

or
$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_0c}(1-\cos\theta)$$

or
$$\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta) \qquad ...(10)$$

$\therefore$ The change in wavelength $= d\lambda = \dfrac{h}{m_0c}(1-\cos\theta)$

This relation shows that $d\lambda$ is independent of the wavelength of the incident radiations as well as the nature of the scattering substance. $d\lambda$ depends upon the angle of scattering only.

Case 1. When $\theta = 0$, $\cos\theta = 1$ and hence $d\lambda = 0$

Case 2. When $\theta = 90°$, $\cos\theta = 0$ and hence

$$d\lambda = \frac{h}{m_0c} = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31}) \times (3 \times 10^8)}\,m = 0.0243 \text{ Å}$$

From Eq. (5), the number of electrons per kilogram is

$$\frac{n}{\rho} = 3 \times 10^{26} \text{ electrons/kg}$$

The number of carbon atoms per kilogram is

$$\frac{N_A}{A} = \frac{6.02 \times 10^{26} \text{ atoms/k mole}}{12 \text{ kg/k mole}} = 5 \times 10^{25} \text{ atoms/kg}$$

$$\left.\begin{array}{l}\text{The number of electrons}\\ \text{per carbon atom}\end{array}\right\} = \frac{3 \times 10^{26}}{5 \times 10^{25}} = 6$$

The ratio of the power scattered to the primary intensity is called the *scattering cross section* (or *scattering coefficient*) of the free electron, designated by σ_e.

$$\sigma_e = \left(\frac{8\pi}{3}\right)\left(\frac{e^2}{4\pi\epsilon_0 mc^2}\right)^2$$

5.13 COMPTON SCATTERING

The Compton Effect. *Compton discovered that when X-rays of a sharply defined frequency were incident on a material of low atomic number like carbon, they suffered a change of frequency on scattering. The scattered beam contains two wavelengths. In addition to the expected incident wavelength, there exists a line of longer wavelength. The change of wavelength is due to loss of energy of the incident X-rays. This elastic interaction is known as Compton effect.*

In the case of incoherent scattering, a scattered beam undergoes not only deviation in its direction but also change of wavelength occurs. In Compton effect, there is a change in wavelength of the scattered beam along with the change in its direction. Hence Compton effect is an incoherent scattering.

This effect was explained by Compton on the basis of quantum theory of radiation. The whole process is treated as a particle collision event between X-ray photon and a loosely bound electron of the scatterer. In this process, both momentum and energy are conserved. In the photon-electron collision, a portion of the energy of the photon is transferred to the electron. As a result, the X-ray proceeds with less than the original energy (and therefore has a lower frequency or a higher wavelength).

The incident photon with an enegy $h\nu$ and momentum $h\nu/c$ strikes an electron at rest. The initial momentum of the electron is zero and its initial energy is only the rest mass energy, m_0c^2. The scattered photon of energy $h\nu'$ and momentum $h\nu'/c$ moves off in a direction inclined at an angle θ to the original direction. The electron acquires a momentum mv and moves at an angle ϕ to the original direction. The energy of the recoil electron is mc^2 (Fig. 5.18).

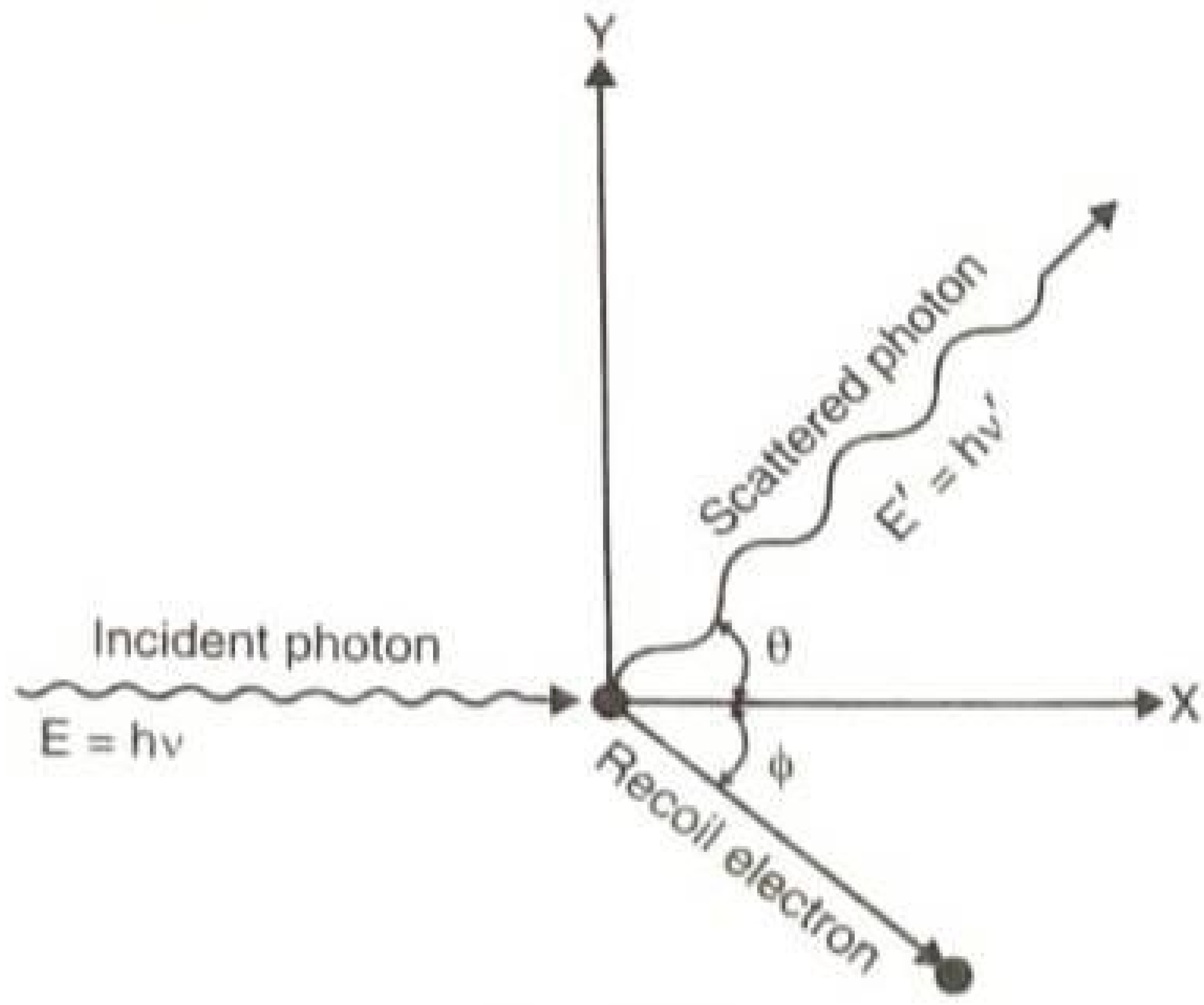


Fig. 5.18

According to the principle of conservation of energy,

$$h\nu + m_0c^2 = h\nu' + mc^2 \quad \text{...(1)}$$

$$I_e = \left(\frac{I}{2}\right)\left[\frac{e^4[1+\cos^2\theta]}{(4\pi\epsilon_0)^2 m^2 c^4 r^2}\right] \qquad ...(8)$$

Let n = number of electrons per unit volume of the scattering medium.

We assume that each electron is equally effective in producing scattered X-rays, irrespective of the presence of the others.

The intensity of the scattered X-rays at the point P, per unit volume of the scatterer is

$$I_s = n I_e$$

$$I_s = \frac{I}{2}\left[\frac{ne^4\ (1+\cos^2\theta)}{(4\pi\epsilon_0)^2 m^2 c^4 r^2}\right] \qquad ...(9)$$

Equation (9) represents Thomson's scattering formula.

5.12 DETERMINATION OF THE NUMBER OF ELECTRONS PER ATOM

Consider a sphere of radius r with centre O (Fig. 5.17). The scatterer is at O.

Take two points A and B on the surface. The element AB subtends an angle $d\theta$ at O.

Surface area of the element

$$= 2\pi\,(AC)\,(AB)$$
$$= 2\pi\,(r\sin\theta)\,r\,d\theta$$
$$= 2\pi\,r^2\sin\theta\,d\theta$$

The total amount of energy scattered per unit volume of the scatterer in unit time is

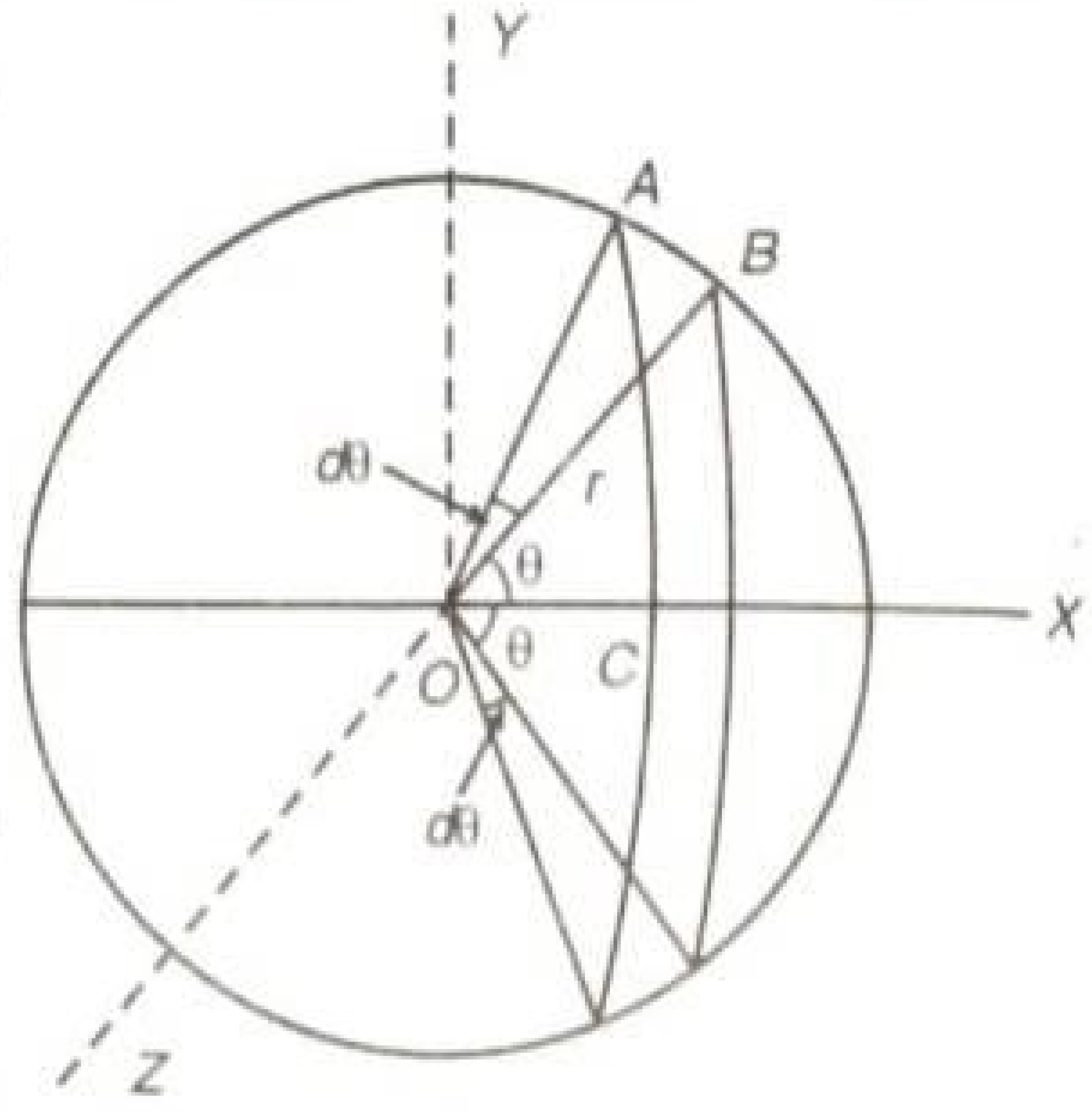


Fig. 5.17

$$U_s = \int_0^{\pi} I_s\,(2\pi r^2\sin\theta)\,d\theta$$

$$U_s = \left(\frac{I}{2}\right)\left[\frac{2\pi n e^4 r^2}{(4\pi\epsilon_0)^2 m^2 c^4 r^2}\right]\int_0^{\pi}(1+\cos^2\theta)\sin\theta\,d\theta \qquad ...(1)$$

$$U_s = \left(\frac{8\pi}{3}\right)\left[\frac{I n e^4}{(4\pi\epsilon_0)^2 m^2 c^4}\right] \qquad ...(2)$$

The linear scattering coefficient σ *of the material is*

$$\sigma = \frac{U_s}{I} = \frac{8\pi}{3}\left[\frac{n e^4}{(4\pi\epsilon_0)^2 m^2 c^4}\right] \qquad ...(3)$$

$$\therefore \quad n = \left[\frac{3}{8\pi}\right]\left[\frac{(4\pi\epsilon_0)^2 m^2 c^4}{e^4}\right]\sigma \qquad ...(4)$$

By measuring σ, *the value of* n can be determined.

n/ρ represents the number of electrons per unit mass.

$$\therefore \quad \frac{n}{\rho} = \left[\frac{3}{8\pi}\right]\left[\frac{(4\pi\epsilon_0)^2 m^2 c^4}{e^4}\right]\left(\frac{\sigma}{\rho}\right) \qquad ...(5)$$

The ratio of σ to the density ρ of the scatterer is the *mass scattering coefficient* σ_m.

Barkla and his coworkers found that the value of σ_m is 0.02 m^2/kg for carbon.

Characteristics of Satellite lines

1. Satellite lines have low intensity.
2. The excitation potential of certain satellites is greater than that of the corresponding "parent" line.
3. Siegbahn has shown that five satellites can be excited by the side of L_{α_1} line of *Mo* by raising the excitation potential. Their appearance corresponds to singly, doubly, trebly etc., ionized atoms, since the intensity of ionisation can be increased by increasing the excitation potential.
4. The intensity of K satellite lines decreases in a continuous manner with increasing atomic number of radiating material. The intensity of L_α satellites decreases abruptly as the atomic number increases from 47 to 50. The intensity again increases abruptly at 75. The L_α satellites are absent between the atomic numbers 50 and 75.
5. If somehow or the other it is possible to avoid multiple ionisation, the satellites are found to be absent.

Explanation of Satellite lines

The explanation of satellites was put forward by Wentzel and Druyvesteyn on the principle of *multiple ionisation of the inner electrons*. It is found in the case of K satellites that the energy of excitation is equal to the energy required to eject a K electron and in addition an L electron from the atom. Hence the initial state for the emission of K satellite is a state of double ionisation. In this case, the atom has an electronic vacancy both in K shell and L shell. Such a state of an atom is designated as KL atomic state. In a similar fashion, other states of double ionisation are designated as KK, KM, LL, LM etc.

An atom in KL atomic state may undergo a radiative transition as $KL \rightarrow KM$ (an electron jumping from M shell to L shell) or $KL \rightarrow LL$ (an electron dropping from L shell to K shell). The estimates of atomic energy indicate that the change in energy in the transition $KL \rightarrow LL$ is greater than that which takes place in the transition $K \rightarrow L$ (the transition which gives rise to K_α line). Hence the former transition gives rise to satellites on the short wavelength side of K_α line. Similarly, the transition $KL \rightarrow LM$ gives rise to satellites on short wavelength side of K_α line. If now we assume that a cathode ray electron ejects two electrons at once from the atom, the probability of this ejection decreases with increasing atomic number of radiating material. This is found to be the case for K series.

5.9 MOSELEY'S LAW

Moseley plotted the square root of the frequencies ($\sqrt{\nu}$) of a given line (say K_α) against the atomic numbers (Z) of the elements emitting that line. Moseley obtained a straight line as shown in Fig. 5.13. The same linear relation was found to hold good for any line in any series. He concluded, therefore, that atomic number (and not atomic weight) is the fundamental property of elements.

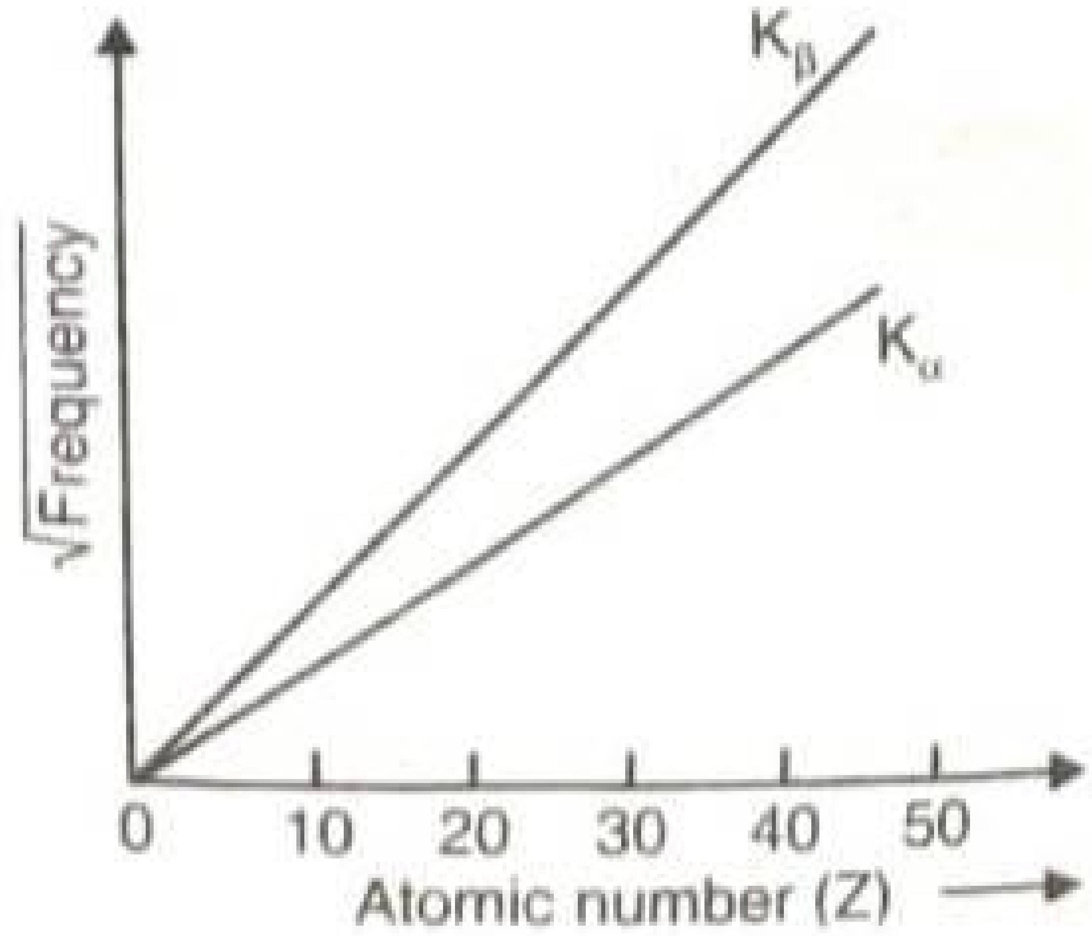


Fig. 5.13

Moseley's law: statement. *The frequency of a spectral line in X-ray spectrum, varies as the square of the atomic number of the element emitting it,* or $\nu \propto Z^2$. Moseley's law may be written as $\sqrt{\nu} = a\,(Z-b)$. Here, Z is the atomic number of the element and a and b are constants depending upon the particular line.

acceleration (a) in the direction of the electric field E.

$$a = eE/m \qquad \text{... (1)}$$

According to classical electromagnetic theory, an accelerated charge radiates electromagnetic waves. The electric vector of the scattered X-rays at a point P with polar coordinates r and ϕ is given by

$$E_\phi = \frac{ea \sin\phi}{4\pi \epsilon_0 c^2 r} = \frac{Ee^2 \sin\phi}{4\pi \epsilon_0 mc^2 r} \qquad \text{...(2)}$$

$$\therefore \quad \frac{E_\phi}{E} = \frac{e^2 \sin\phi}{4\pi \epsilon_0 mc^2 r} \qquad \text{...(3)}$$

Intensity is proportional to the square of the electric vector.

So the ratio of the scattered intensity at P to the incident intensity is

$$\frac{I_\phi}{I} = \left(\frac{E_\phi}{E}\right)^2 = \frac{e^4 \sin^2\phi}{(4\pi\epsilon_0)^2 m^2 c^4 r^2} \qquad \text{...(4)}$$

Consider a case where the incident beam is along the X-axis and its electric vector E is in the YZ plane (Fig. 5.16). Resolve E into two perpendicular components along the Y and Z axes.

$$E^2 = E_y^2 + E_z^2$$

$$\therefore \quad I = I_y + I_z$$

Here I_y and I_z are the components of the wave along the Y and Z axes.

The incident beam is unpolarized.

$$\therefore \quad I_y = I_z = \frac{I}{2} \qquad \text{...(5)}$$

Let I_1 and I_2 be the intensities of the scattered beam at the point P due to the y and z components.

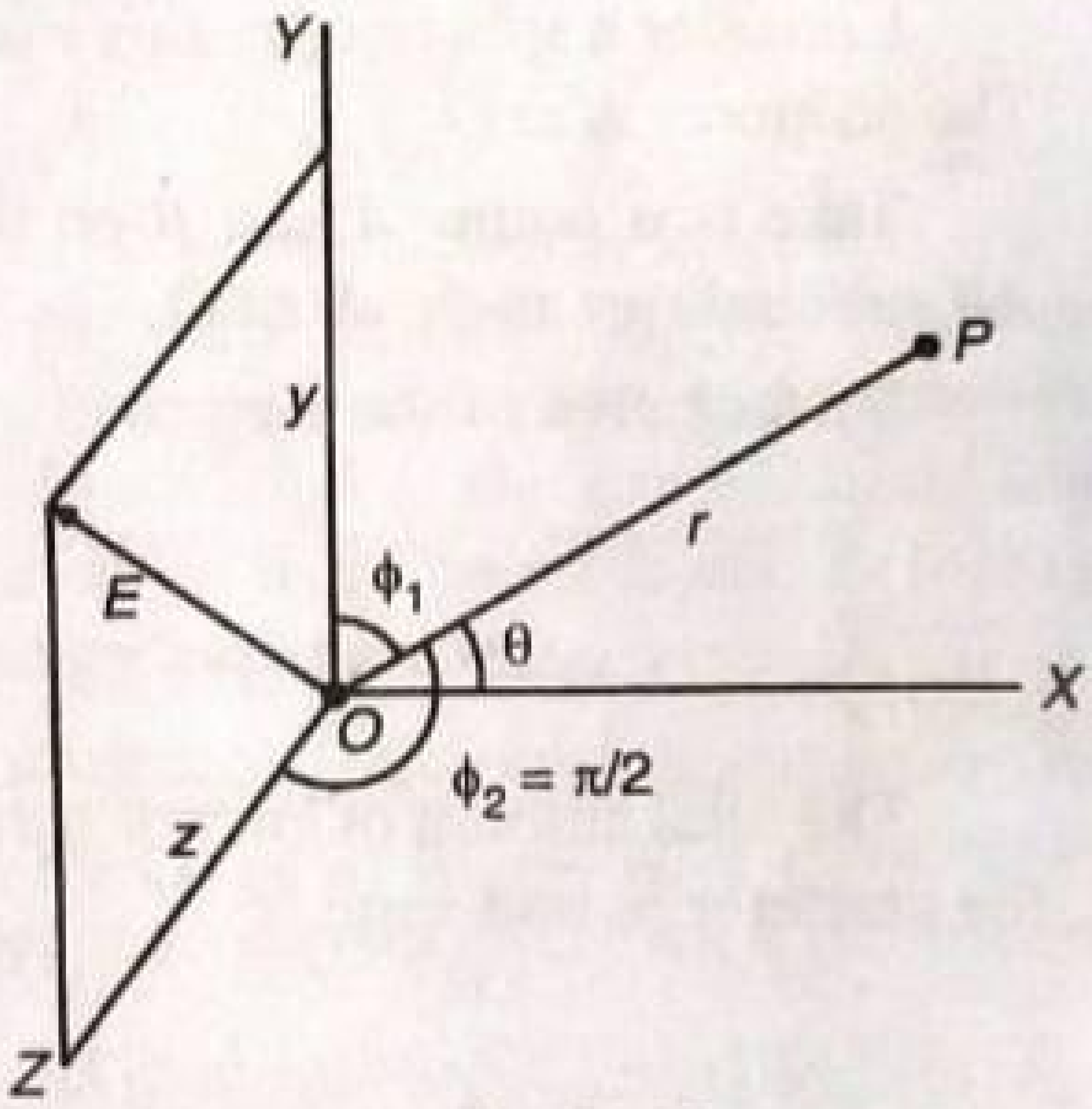


Fig. 5.16

$$\therefore \quad I_1 = I_y \left[\frac{e^4 \sin^2\phi_1}{(4\pi\epsilon_0)^2 m^2 c^4 r^2}\right]$$

$$I_y = \frac{I}{2} \text{ and } \sin^2\phi_1 = \cos^2\theta$$

Here θ is the angle between r and the x-axis.

$$\therefore \quad I_1 = \frac{I}{2}\left[\frac{e^4 \cos^2\theta}{(4\pi\epsilon_0)^2 m^2 c^4 r^2}\right] \qquad \text{... (6)}$$

Similarly
$$I_2 = I_z \left[\frac{e^4 \sin^2\phi_2}{(4\pi\epsilon_0)^2 m^2 c^4 r^2}\right]$$

But
$$I_z = \frac{I}{2} \text{ and } \sin^2\phi_2 = 1.$$

Here the angle ϕ_2 between OP and OZ is always $\pi/2$.

$$\therefore \quad I_2 = \left(\frac{I}{2}\right)\left[\frac{e^4}{(4\pi\epsilon_0)^2 m^2 c^4 r^2}\right] \qquad \text{... (7)}$$

The total intensity of the scattered wave at P is given by

$$I_e = I_1 + I_2$$

Scanned by TapScanner

Most of the electrons that generate X-ray photons give up only a part of their energy in this way. Therefore, most of the X-radiation is of longer wavelength than λ_{min}. Thus, the continuous spectrum is the result of the inverse photoelectric effect, with electron kinetic energy (eV) being transformed into photon energy ($h\nu$).

5.6 CHARACTERISTIC X-RAY SPECTRUM

There are two methods of producing characteristic X-rays.

(1) The characteristic X-rays of an element can be excited by using the element as the target in the X-ray tube and thus subjecting it to direct bombardment by electrons. For each target there is a minimum potential below which the line spectra do not appear. This critical P.D. below which the line spectra do not appear, is different for different targets. Molybdenum shows up the line spectra only if the P.D. is above 35 kV.

(2) Characteristic X-rays of an element can also be excited by allowing primary X-rays from a hard X-ray tube to fall on the element. The primary X-rays must be harder than the characteristic X-rays to be produced.

The peaks obtained in the X-ray spectrum (Fig. 5.12) give us the line spectrum which is characteristic of the element used in the target. The group of lines of shortest wavelength is called the K-series. Usually two lines of this series are detected. These lines are termed as K_α and K_β lines in the order of decreasing wavelengths. The next group is called the L-series of longer wavelengths (L_α, L_β, L_γ, etc.). For heavier elements a third series, called the M-series has been detected.

Origin of characteristic X-rays. This can be understood in terms of Bohr's theory. Suppose an atom in the target of an X-ray tube is bombarded by a high-speed electron and a K-electron is removed. A vacancy is created in the K-shell. This vacancy can be filled up by an electron from either of L, M or N shells or a free electron. These possible transitions can result in the K_α, K_β lines and the limiting line. Similarly, the longer wavelength L-series originates when an L electron is knocked out of the atom, the M-series when an M electron is knocked out and so on (Fig. 5.11).

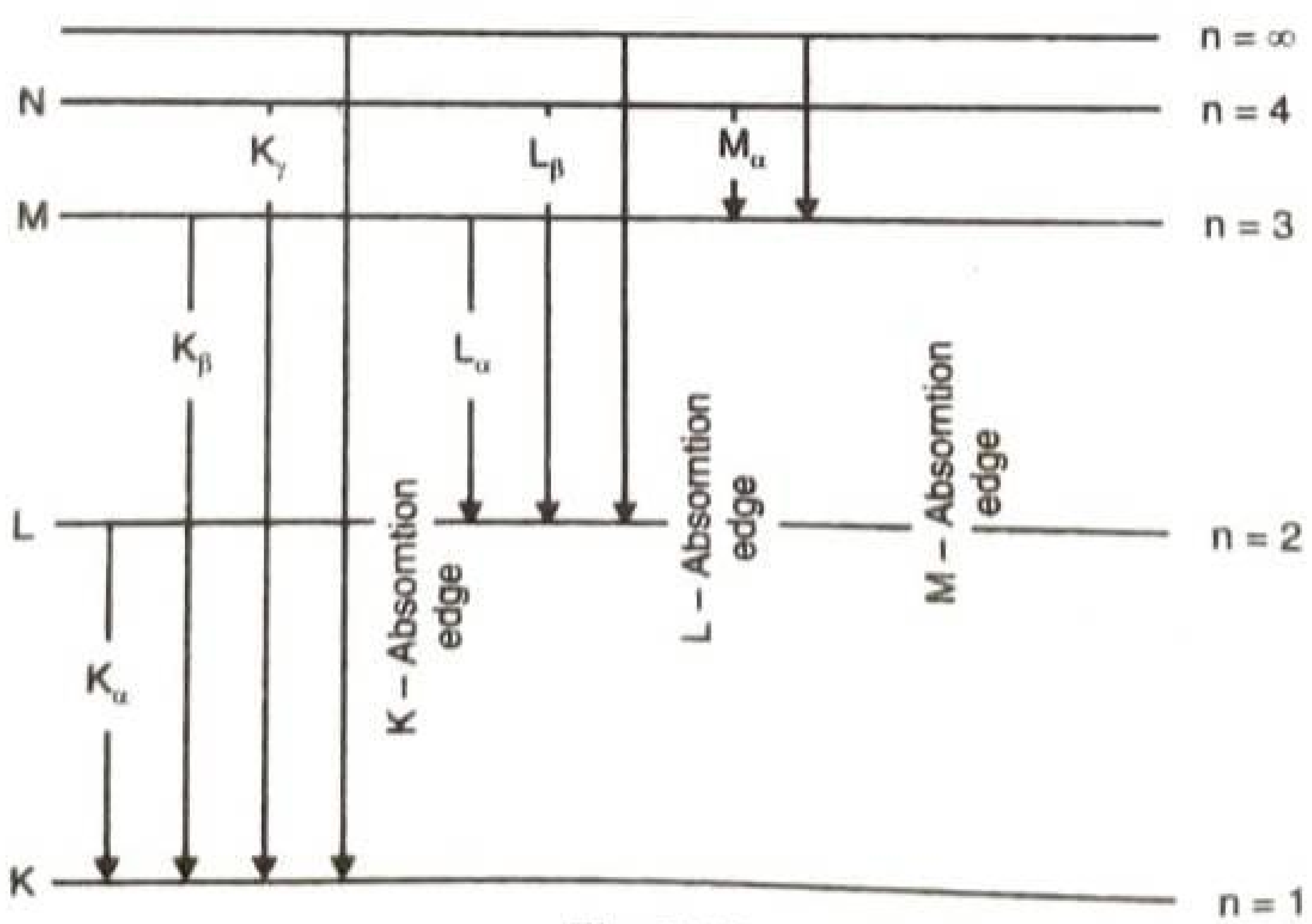


Fig. 5.11

It is clear that continuous spectra and line spectra both emitted by the same target are of different origin. The continuous spectrum is the result of the inverse photoelectric effect, with electron K.E.

being transformed into photon energy $h\nu$. The line spectrum has its origin in electronic transitions within atoms that have been disturbed by the incident electrons.

Fig. 5.12 shows a characteristic spectrum superimposed upon continuous spectrum.

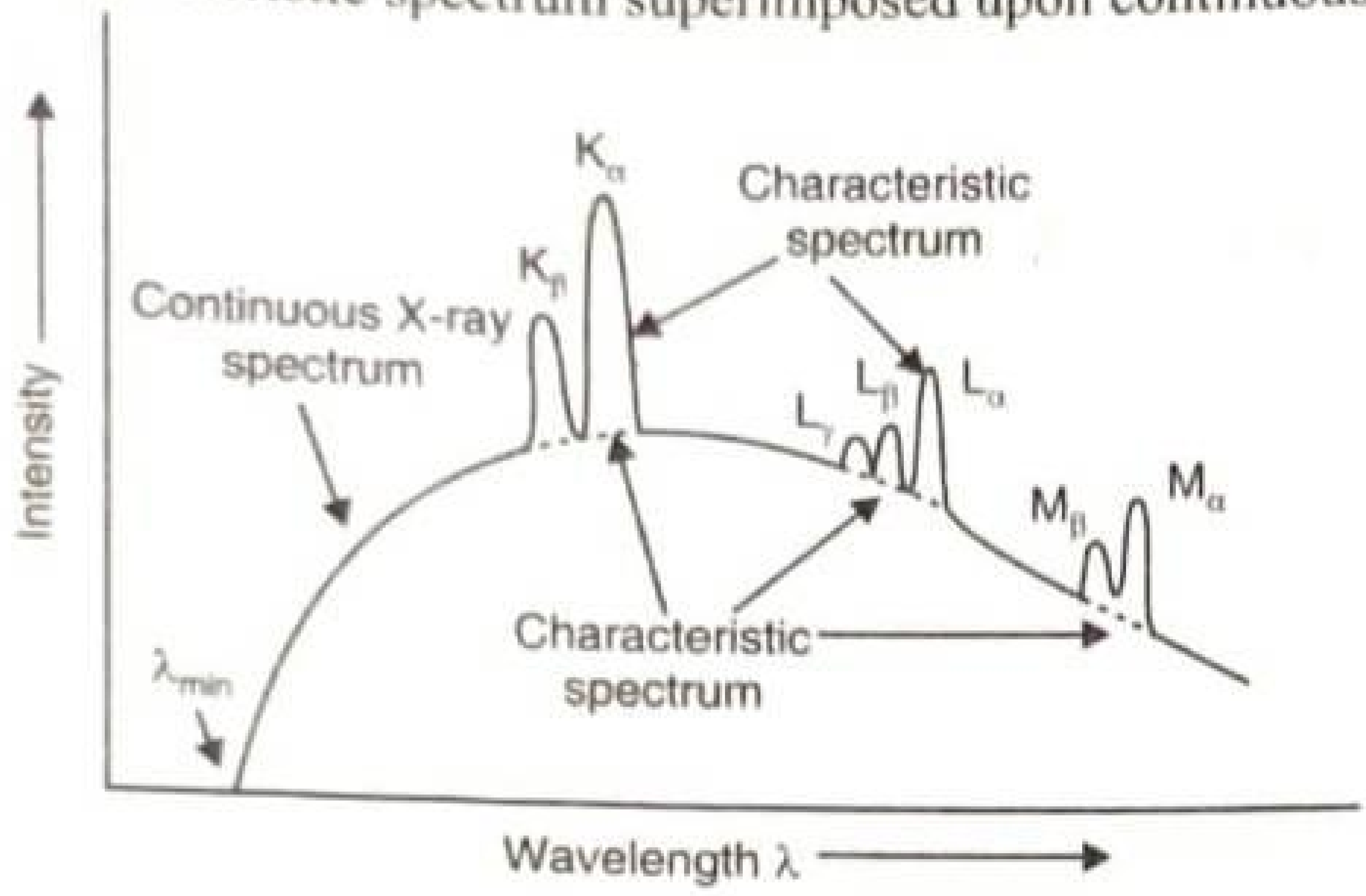


Fig. 5.12

5.7 AUGER EFFECT

We have seen that when a K-electron is knocked out from an atom either by high-energy electron impact or through absorption of an X-ray photon, the vacancy thus created in the K-shell is filled up by transition from an outer-shell (say, L) electron. The energy of the emitted characteristic x-ray photon is $h\nu_{KL} = E_K - E_L$.

But Auger found that there are certain radiationless transitions.

The excess energy in such transitions may be directly absorbed by another L-electron which gets emitted. There are now two vacancies created in L-shell and the energy level of the atom is different from L-level. If we designate this as LL-level of energy E_{LL}, then the kinetic energy of the emitted second electron is given by

$$\frac{1}{2}mv^2 = E_K - E_{LL}$$

Such a transition in which there is no emission of electromagnetic radiation, but emission of two electrons from the same atom is called **Auger transition** or **Auger effect**. It is also called *radiationless transition*.

Thus, the de-excitation of the atom may be accompanied either by the emission of a photon (characteristic radiation) or an electron (Auger electron).

5.8 SATELLITES

- Moseley and his contemporaries observed the X-ray spectral lines which were intense and easily resolvable. These lines are due to transitions between states of *single ionization* and can be easily fitted on an energy level diagram. Therefore, they are called *diagram lines*.
- Later on, with the improved resolving power of the instruments, many more lines were discovered. Most of them were rather faint and were usually found close to and on the short wavelength side of the more intense lines. Hence they were called "satellites". They could not be fitted in the energy level diagram. Hence the new lines are called *nondiagram lines*.

5.5 X-RAY SPECTRA

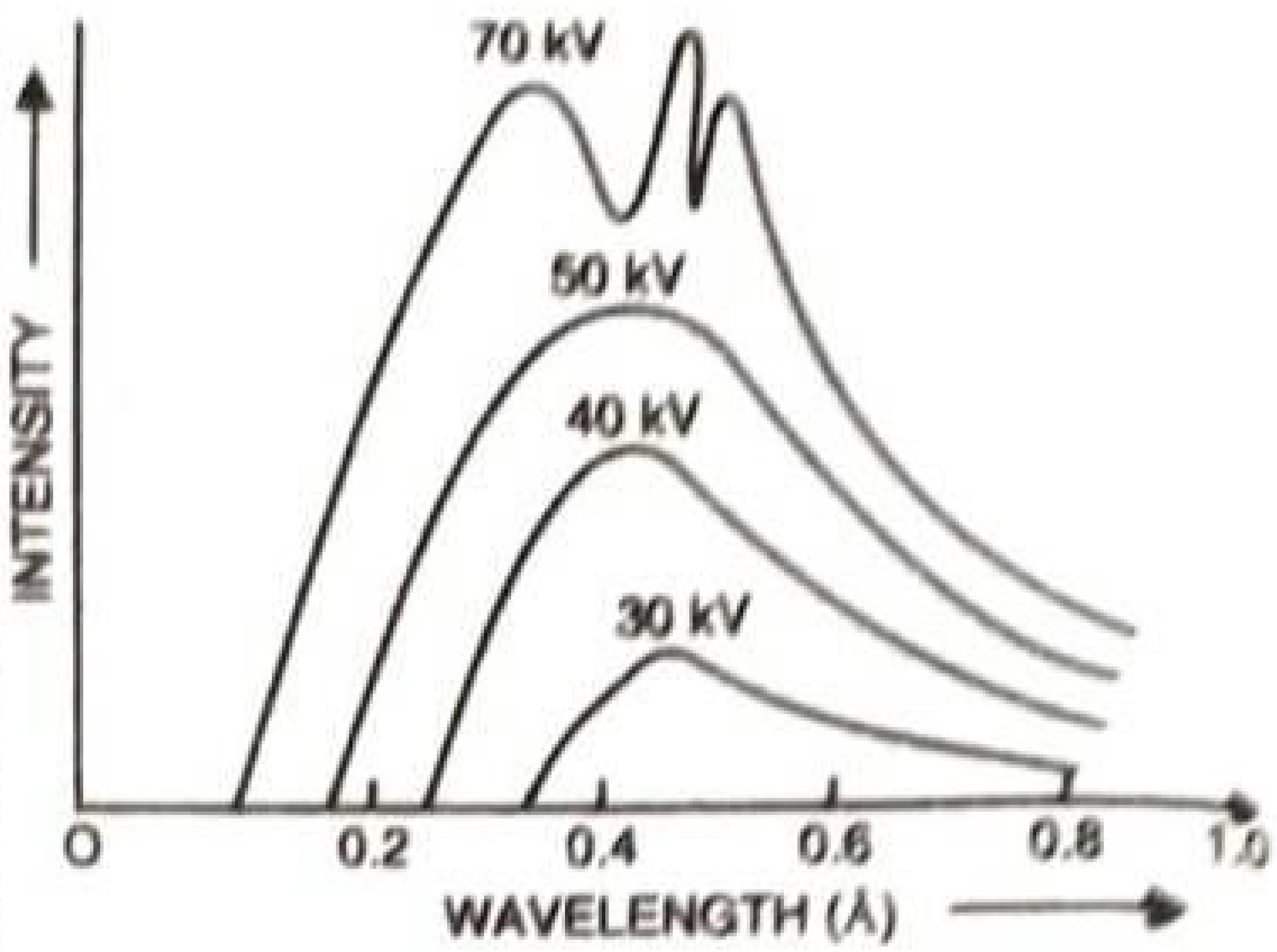


Fig. 5.9

Urey and his co-workers analysed the X-ray beam emitted from an X-ray tube using different potential differences and same target. Using tungsten as the target and different potential differences, the intensities of the rays produced are plotted against wavelength. Fig. 5.9 represents these graphs. For applied P.D.s 30 *kV*, 40 *kV* and 50 *kV*, the spectrum is *white*. But, for the applied P.D. 70 *kV*, two sharp peaks are seen. The sharp peaks show the *line* or *characteristic radiation*. The line spectra are absent, till the P.D. is greater than a particular value. The smoothly varying curves represent the *continuous spectrum*. The superimposed lines on the continuous background constitute the *characteristic spectrum*.

Main features of continuous X-ray spectrum

(1) For each anode potential, there is a minimum wavelength (λ_{min}) below which no radiation is emitted. Above this critical value, the intensity of the radiation increases rapidly with increasing wavelengths and after reaching a maximum, decreases gradually. The intensity never reaches zero showing that the radiation contains all possible wavelengths above the minimum limit.

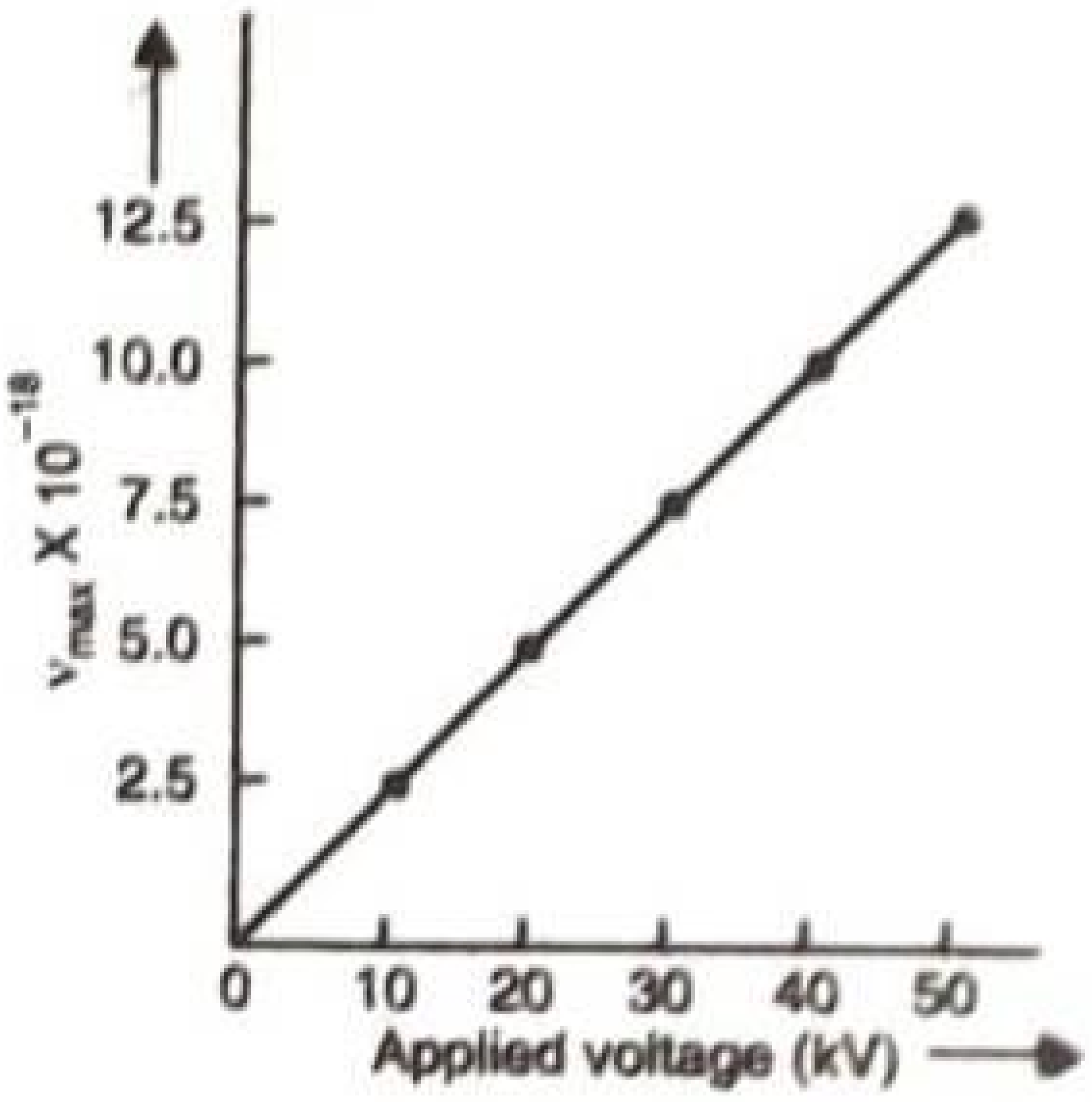


Fig. 5.10

(2) When the voltage across the X-ray tube is increased, λ_{min} is shifted towards smaller values. Duane and Hunt showed that λ_{min} is inversely proportional to the applied voltage V or ν_{max} is directly proportional to V. If the limiting frequencies (ν_{max}) are plotted against the applied voltages (V), a straight line graph passing through the origin is obtained (Fig. 5.10). This empirical law of Duane and Hunt is expressed analytically as

$$eV = h\nu_{\max} = \frac{hc}{\lambda_{\min}}.$$

Duane-Hunt Law: Statement. *The short wavelength limit (λ_{min}) of the X-ray spectrum is inversely proportional to potential (V) applied across the tube terminals.*

$$\lambda_{\min} \propto \frac{1}{V}.$$

Explanation. eV is the K.E. of the bombarding electron. If the entire K.E. of the electron striking the target is converted into the energy of the X-ray photon, then, $eV = h\nu_{max}$ according to Einstein's theory.

But $$\nu_{max} = \frac{c}{\lambda_{\min}}$$

$\therefore$ $$eV = \frac{hc}{\lambda_{\min}} \quad \text{or} \quad \lambda_{\min} = \frac{hc}{eV} \quad \text{or} \quad \lambda_{\min} \propto \frac{1}{V}.$$

longer path than the ray PQR. To compute the path-difference between the two rays, from Q draw normals QT and QS on $P'Q'$ and $Q'R'$ respectively.

$$\text{Path-difference} = TQ' + Q'S = d\sin\theta + d\sin\theta = 2d\sin\theta.$$

Hence the two rays will reinforce each other and produce maximum intensity, if

$$2d\sin\theta = n\lambda.$$

Here, $n = 1, 2, 3......$ The integer n gives the order of the scattered beam, λ is the wavelength of the X-rays used. This equation is called *Bragg's law.*

5.4 THE BRAGG X-RAY SPECTROMETER

Experimental arrangement. Fig. 5.6 shows the essential parts of a Bragg spectrometer.

It consists of three parts. (1) a source of X-rays (2) a crystal held on a circular table which is graduated and provided with vernier and (3) a detector (ionisation chamber).

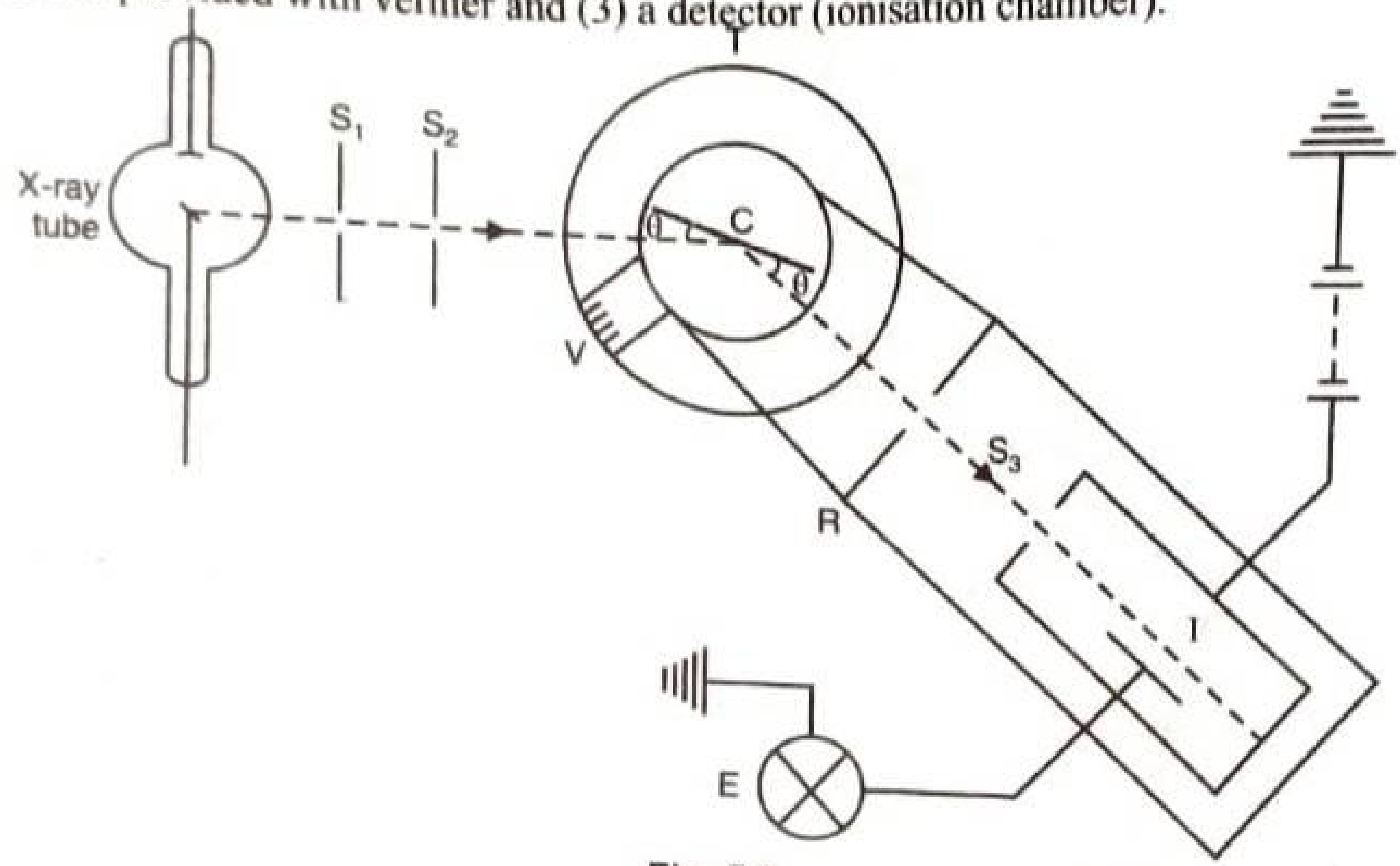


Fig. 5.6

- X–rays from an X–ray tube, limited by two narrow lead slits S_1 and S_2, are allowed to fall upon the crystal C.
- The crystal is mounted on the circular table T, which can rotate about a vertical axis and its position can be determined by the vernier V.
- The table is provided with a radial arm (R) which carries an ionisation chamber (I). This arm also can be rotated about the same vertical axis as the crystal. The position of this arm can be determined by a second vernier (not shown in the figure). The ionisation chamber is connected to an electrometer (E) to measure the ionisation current. Hence we can measure the intensity of the diffracted beam of X–rays, diffracted in the direction of the ionisation chamber. S_3 is a lead slit, to limit the width of the diffracted beam. In practice, the crystal table is geared to the ionisation chamber so that the chamber turns through 2θ when the crystal is turned through θ.

Working. To begin with, the glancing angle θ for the incident beam is kept very small. The ionisation chamber is adjusted to receive the reflected beam till the rate of deflection is maximum. The glancing angle (θ) and the intensity of the diffracted beam (I) are measured. The glancing angle is next increased in equal steps, by rotating the crystal table. The ionisation current is noted for

X-rays are collimated by slits, rendered monochromatic by a Bragg reflection, and passed through the material under study. The thickness of the absorbing material is varied and the transmitted intensity is plotted against thickness (Fig. 5.3). This is an exponential decay curve ($I = I_0 e^{-\mu x}$).

5.2.1. X-ray Absorption Edges

The value of the linear absorption coefficient μ depends upon the X-ray wavelength and on the nature of the absorbing material. Let X-ray photons of sufficiently high energy (short wavelength) be made to fall upon a material. A major part of the incident energy of X-ray photons is spent in ejecting electrons from the *K*-shell of the atom. The energy of the X-ray beam is thus absorbed photo electrically, causing the ejection of the *K*-electrons. But if the incident radiation has a wavelength slightly longer (or the energy slightly smaller) than that required to eject the *K*-electrons, there is no photo-electric absorption. At this stage, there is a sudden fall in the value of the absorption coefficient (Fig. 5.4).

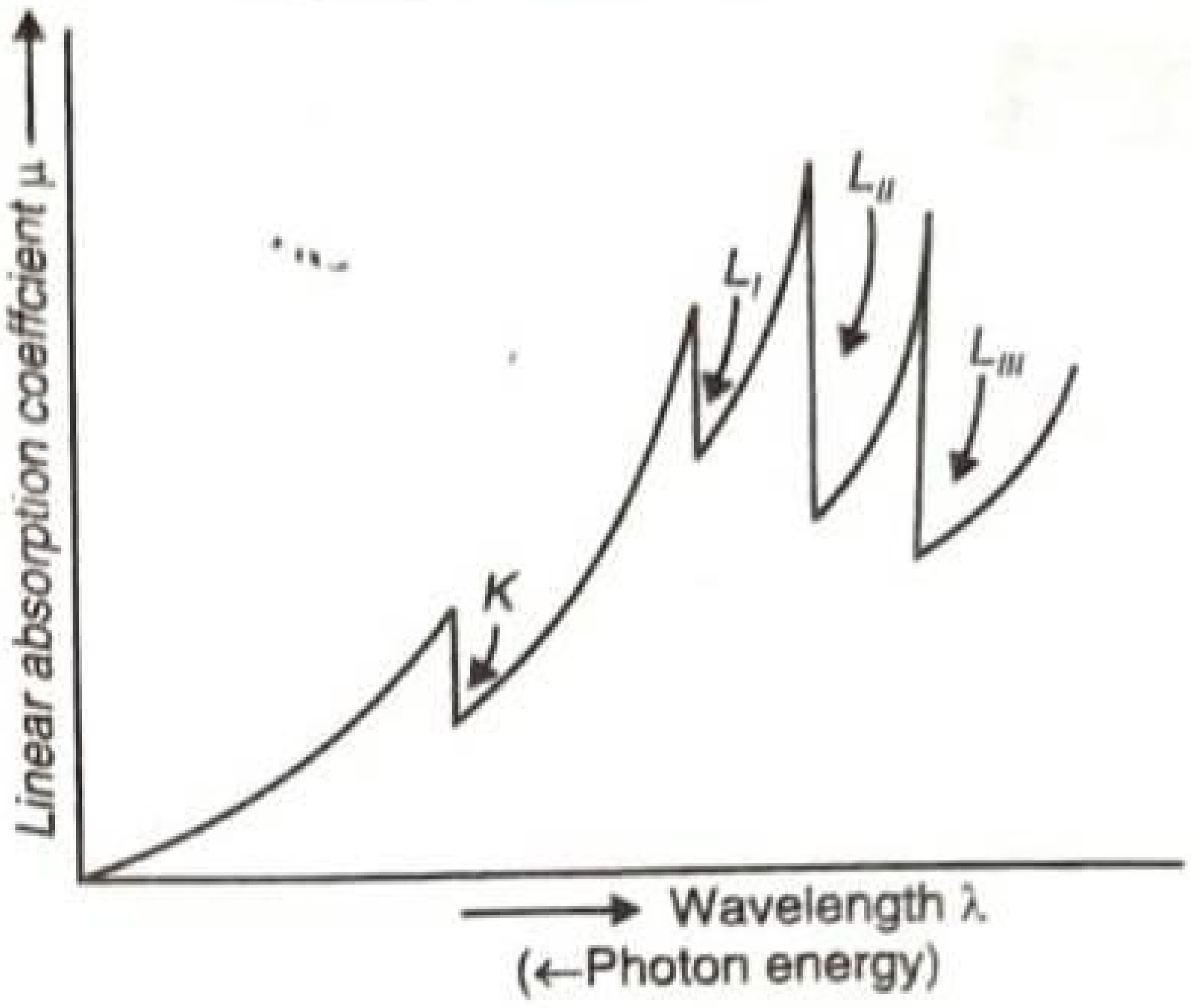


Fig. 5.4

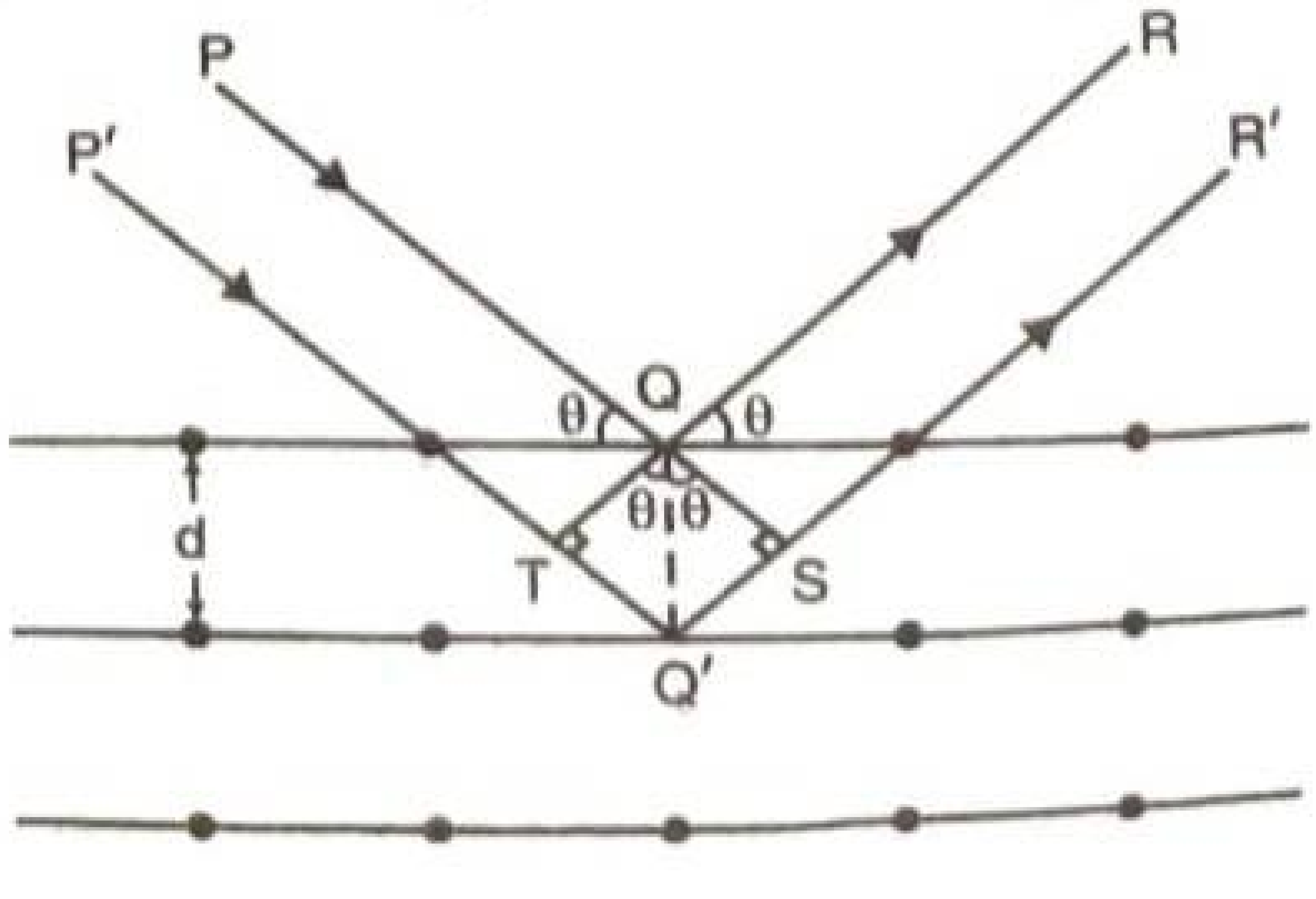
A wavelength at which there is a sudden change in the absorption coefficient of a given material for the X-*ray beam is called* **absorption edge.**

As the wavelength is further increased, the absorption coefficient again increases. The X-ray photons now start ejecting electrons from the *L*-shell. There is again a sudden decrease in the value of μ at another definite wavelength $\lambda = \lambda_{LI}$. Immediately after this there are two further discontinuities at two closely lying wavelengths $\lambda_{L\,II}$ and $\lambda_{L\,III}$. In the case of ionization from the *L*-shell there are three absorption edges (L_I, L_{II}, L_{III}), indicating that the electrons in the *L*-shell can exist in three energy sub-group states.

5.3 BRAGG'S LAW

When monochromatic X-rays impinge upon the atoms in a crystal lattice, each atom acts as a source of scattering radiation of the same wavelength. The crystal acts as a series of parallel reflecting planes. The intensity of the reflected beam at certain angles will be maximum when the path difference between two reflected waves from two different planes is an integral multiple of λ.

Derivation of Bragg's law. Consider a set of parallel planes of atom points at a spacing *d* between two successive planes. Let a narrow monochromatic X-ray beam of wavelength λ be incident on the first plane at a glancing angle θ (Fig. 5.5). Consider the ray *PQ* incident on the first plane. The corresponding reflected ray *QR* must also be inclined at the same angle θ to the plane. Since X-rays are much more penetrating than ordinary light, there is only partial reflection at each plane. The complete absorption takes place only after penetrating several layers.



- Consider two parallel rays *PQR* and *P′Q′R′* in the beam, which are reflected by two atoms *Q* and

5.1 PRODUCTION OF X-RAYS

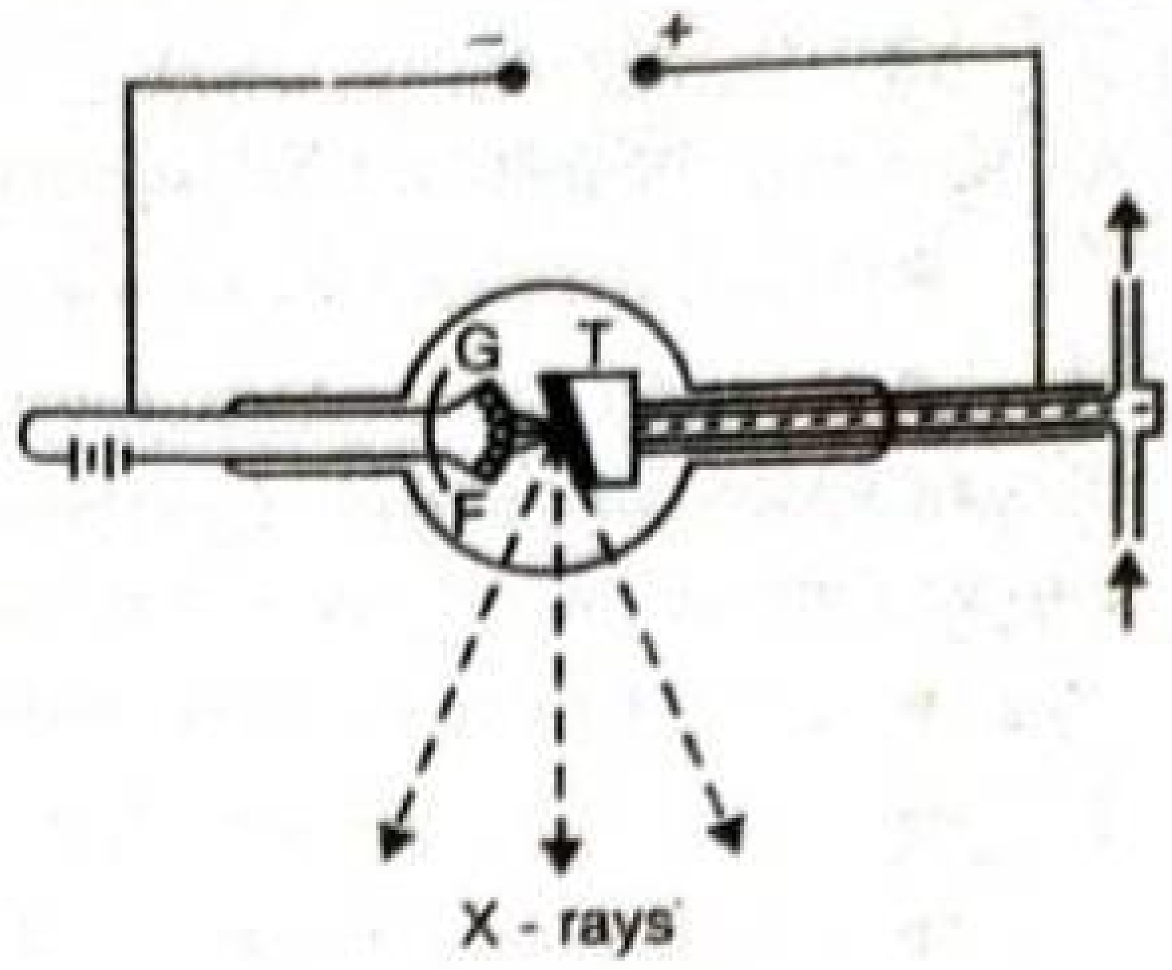


Fig. 5.1

The Coolidge tube. X-rays are produced when fast moving electrons are suddenly stopped by a solid target. A Coolidge tube is shown in Fig. 5.1. The tube is exhausted to the best possible vacuum of the order of 10^{-5} *mm* of mercury. The cathode consists of a tungsten filament (F) heated by a low tension battery. Thermionic electrons emitted by the filament are accelerated towards the target (T) by a high P.D. maintained between F and T. The filament is placed inside a metal cup G to focus the electrons on to the target. The target must be cooled to remove the heat generated in it by continuous electron-bombardment. The usual method is to mount the target material on a hollow copper tube through which cold water is continuously circulated. The target is made of a metal like tungsten or molybdenum having a high melting point and a high atomic number. Metals with high atomic number give more energetic and intense X-rays when used as targets.

Control of intensity and quality. In the Coolidge tube, the intensity and frequency of X-rays can be easily controlled.

(1) The intensity of X-rays depends on the number of electrons striking the target per second. The number of electrons given out by the filament is proportional to its temperature, which can be adjusted by varying the current in the filament circuit. Therefore, the intensity of X-rays varies with the filament current.

(2) The frequency of X-rays emitted depends on the voltage between the cathode and the anode (target).

- Let V be the accelerating potential across the tube.
- e is the charge on the electron.

$$\left.\begin{array}{l}\text{Work done on the electron in moving}\\ \text{from the cathode to the anticathode}\end{array}\right\} = eV.$$

The electron thus acquires K.E. which is converted into X-rays, when the electron strikes the target.

- If ν_{max} is the maximum frequency of the X-rays produced, then

$$h\,\nu_{max} = eV.$$

$\therefore$ The minimum wavelength produced by an X-ray tube $= \lambda_{min} = \dfrac{c}{\nu_{max}} = \dfrac{hc}{eV}.$

- The minimum wavelength of the X-rays for a given voltage V across an X-ray tube is

$$\lambda_{min} = \frac{hc}{eV}$$

$$\lambda_{min} \propto \frac{1}{V}.$$

This is **Duane-Hunt law.**

- X-rays are electromagnetic waves of short wavelengths in the range of 1 nm to 0.05 nm. The longer wavelength end of the spectrum is known as the "*soft* X-rays" and the shorter wavelength end is known as "*hard* X-rays."

5.2 THE ABSORPTION OF X-RAYS

All materials through which X–rays pass absorb them to some extent. If a sheet of any substance is interposed in the path of a homogeneous beam of X–rays, its intensity decreases.

- High frequency X-rays are absorbed less and are called *hard* X-rays.
- Low frequency X-rays are absorbed more and are called *soft* X-rays.

Intensity of X-rays. *The intensity of X-ray beam, at any point, is defined as the amount of energy carried per second per unit area, perpendicular to the direction of flow of energy.*

- Let I_0 be the intensity of the incident X–ray beam.
- Let I be the intensity of the beam after it has traversed a thickness dx of the absorber.
- The decrease in intensity (dI) hollows the equation, $dI = -\mu I dx$.

Here μ is called the linear absorption coefficient which depends on the wavelength of X-ray used and the nature of the absorbing material. μ has the dimensions of reciprocal length (L^{-1}) and its units are therefore m^{-1}.

Now, $$\frac{dI}{I} = -\mu\, dx$$

$\therefore$ $$I = I_0 e^{-\mu x} \qquad ...(1)$$

If $$x = \frac{1}{\mu}, I = \frac{I_0}{e}.$$

The linear absorption co-efficient of the material is, therefore, defined as the reciprocal of the thickness after which the intensity of X-rays falls to 1/e of its original intensity.

Eq. (1) may be put in the form

$$I = I_0 e^{-(\mu/\rho) \times \rho x} = I_0 e^{-\mu_m m}$$

$\mu/\rho = \mu_m$, is called the mass absorption coefficient and m is the mass of unit area of the absorbing sheet. Theory shows that the mass absorption coefficient (μ_m) varies as the cube of the incident wavelength (λ) and also the atomic number (Z) of the absorber. Thus, $\mu_m = k\lambda^3 Z^3$. This explains why materials with high atomic numbers are preferred for shielding against X-rays.

Experimental study. An ionization chamber is used to study the penetrating ability of X-rays with apparatus shown in Fig. 5.2.

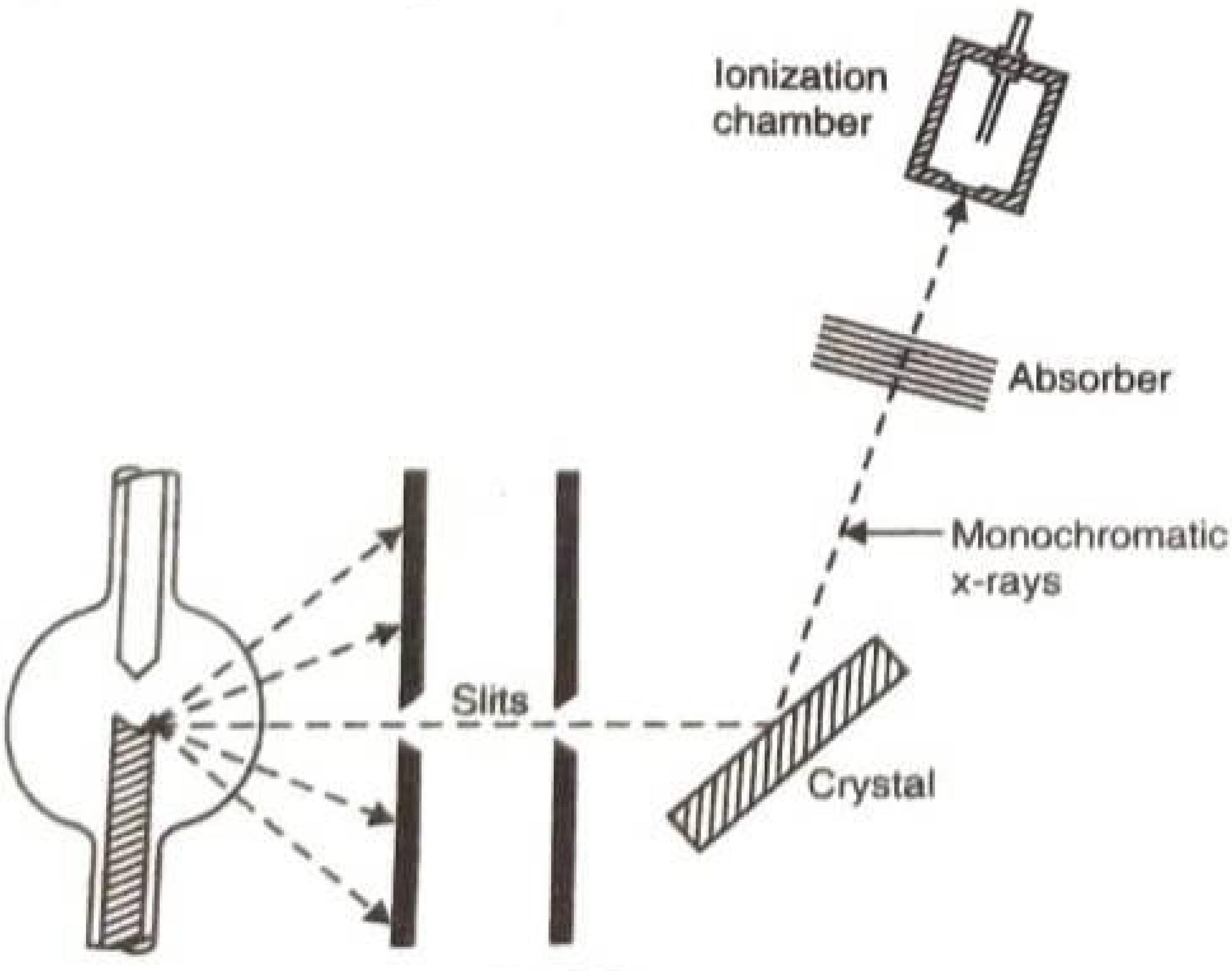


Fig. 5.2

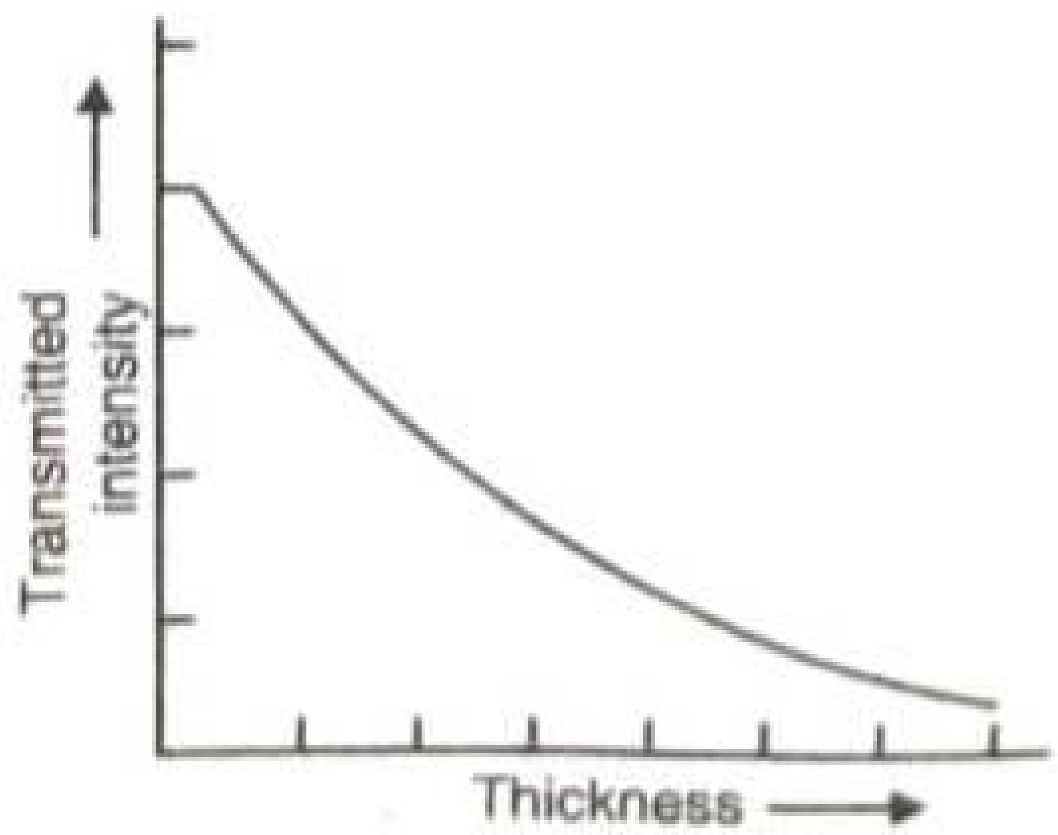


Fig. 5.3

9.51 CRITERION FOR RESOLUTION ACCORDING TO LORD RAYLEIGH

To express the resolving power of an optical instrument as a numerical value, Lord Rayleigh proposed an arbitrary criterion. **According to him, two nearby images are said to be resolved if the position of the central maximum of one coincides with the first secondary minimum of the other and vice versa. The same criterion can be conveniently applied to calculate the resolving power of a telescope, microscope, grating, prism, etc.**

In Fig. 9.63, A and B are the central maxima of the diffraction patterns of two spectral lines of wavelengths λ_1 and λ_2. The difference in

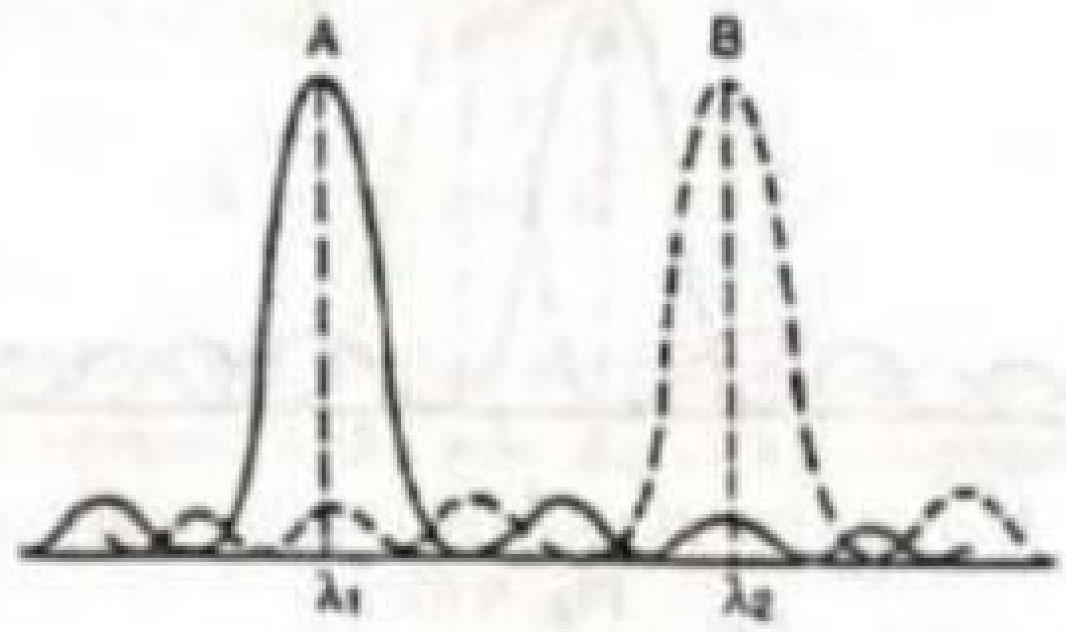


Fig. 9.63

the angle of diffraction is large and the two images can be seen as separate ones. The angle of diffraction corresponding to the central maximum of the image B is greater than the angle of diffraction corresponding to the first minimum at the right of A. Hence the two spectral lines will appear well resolved.

In Fig. 9.64 the central maxima corresponding to the wavelengths λ and $\lambda + d\lambda$ are very close. The angle of diffraction corresponding to

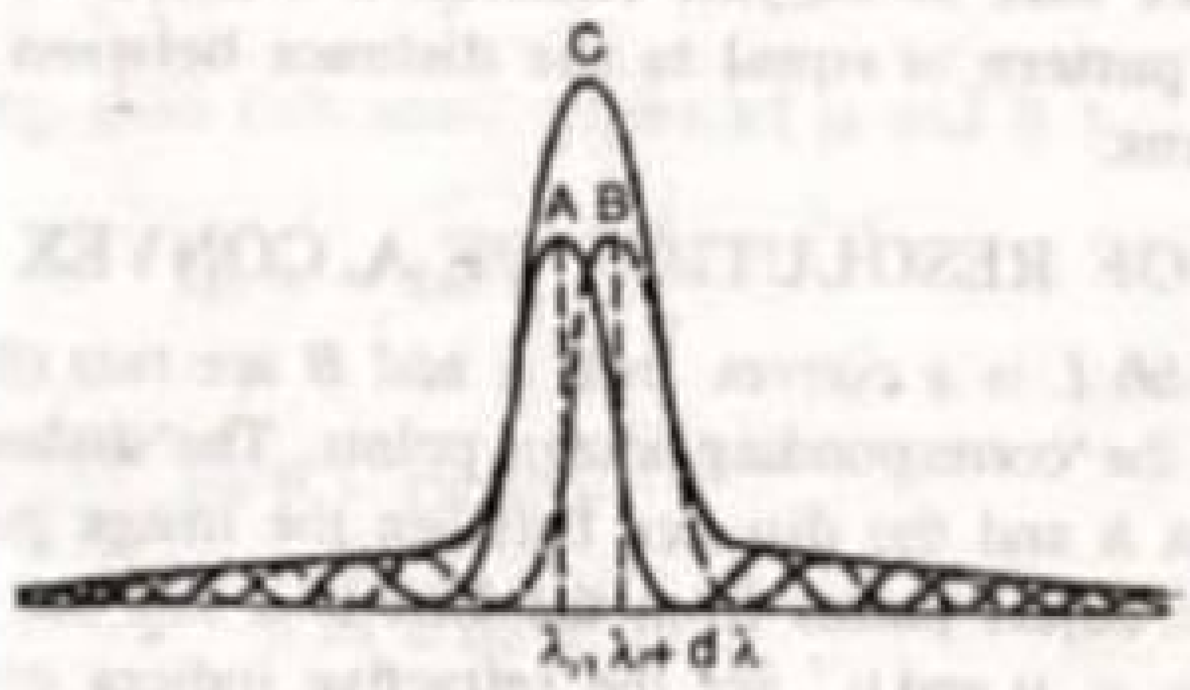
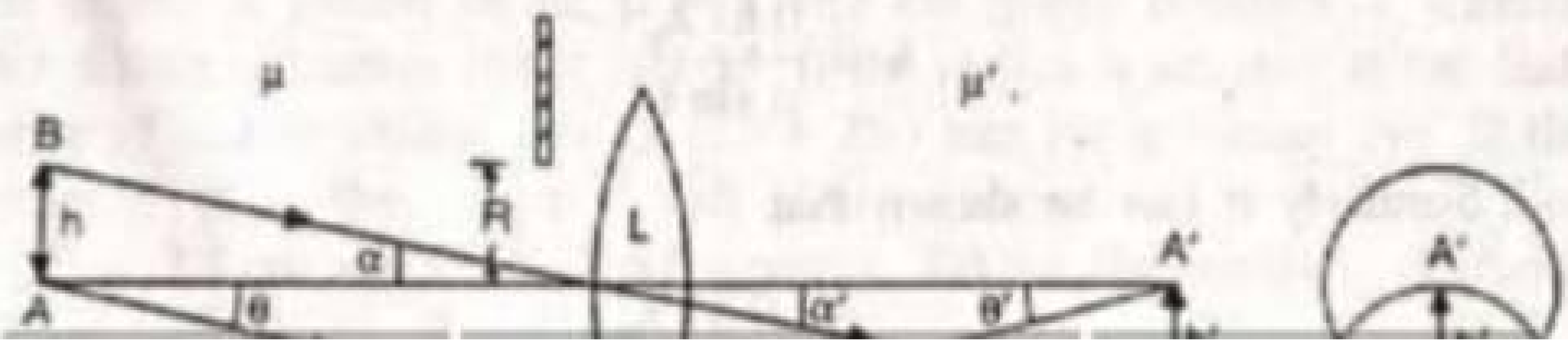


Fig. 9.64

(D is the diameter of the aperture). In the side figure, A' and B' are the centres of the central bright discs of the diffraction patterns of A and B.



are seen as separate ones. **The ability of an optical instrument, expressed in numerical measure, to resolve the images of two nearby points is termed as its resolving power.**

In the case of a prism or a grating spectrograph, the term resolving power is referred to the ability of the prism or grating to resolve two nearby spectral lines so that the two lines can be viewed or photographed as separate lines.

the first minimum of A is greater than the angle of diffraction corresponding to the central maximum of B. Thus, The two images overlap and they cannot be distinguished as separate images. The resultant intensity curve gives a maximum as at C and the intensity of this maximum is higher than the individual intensities of A and B. Thus when the spectrograph is turned from A to B, the intensity increases, becomes maximum at C and then decreases. In this case, the two spectral lines are **not resolved.**

In Fig. 9.65, the position of the central maximum of A (wavelength λ) coincides with the position of the first minimum of B. (wavelength $\lambda + d\lambda$). Similarly, the position of the central maximum of B coincides with

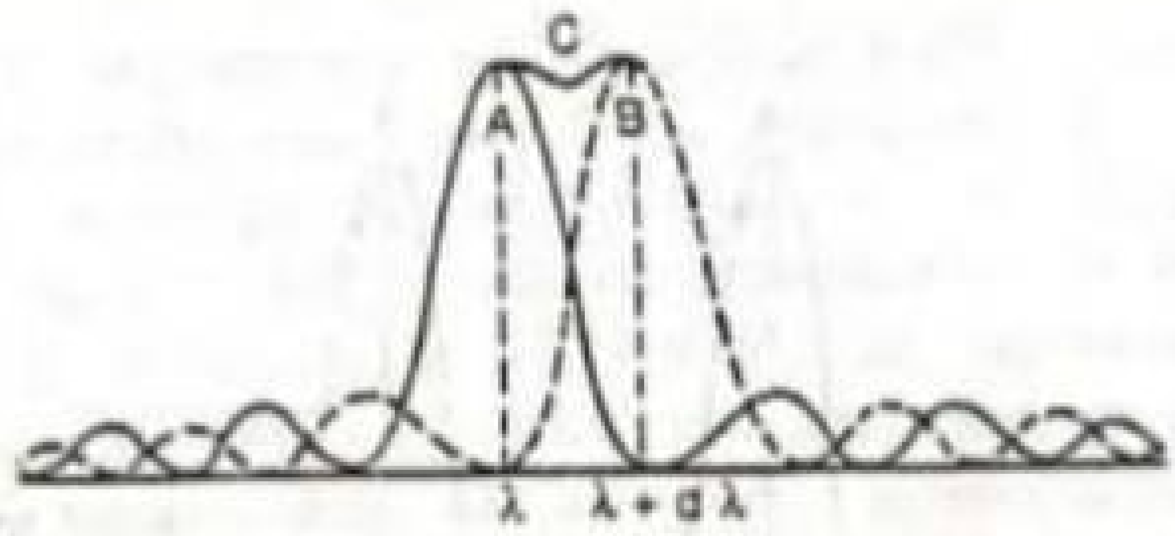


Fig. 9.65

the position of the first minimum of A. Further, the resultant intensity curve shows a **dip** at C *i.e.*, in the middle of the central maxima of A and B (Here, it is assumed that the two spectral lines are of the same intensity). The intensity at C is approximately 20% less than that at A or B. If a spectrograph is turned from the position corresponding to the central image of A to the one corresponding to the image of B, there is noticeable decrease in intensity between the two central maxima. The spectral lines can be distinguished from one another and according to Rayleigh they are said to be **just resolved. Rayleigh's condition can also be stated as follows. Two images are said to be just resolved if the radius of the central disc of either pattern is equal to the distance between the centers of the two patterns.**

9.52 LIMIT OF RESOLUTION OF A CONVEX LENS

In Fig. 9.66 L is a convex lens. A and B are two object points and A' and B' are the corresponding image points. The distance between the object points is h and the distance between the image points is h'. The distance of the object points from the lens is u and the distance of the image points is v. μ and μ' are the refractive indices o[illegible] nd image media. R is the radius of the aperture kept in [illegible]

$$h' = \frac{1}{1000} \text{ cm} = \frac{1}{100} \text{ mm approximately}$$

0.61λ